

Effetti relativistici nel volo spaziale: teorie ed esperimenti

I.I.S. A. Volta Pavia
XXXVII CORSO DI AGGIORNAMENTO IN FISICA - ANNO 2014
Relatività

8 Ottobre 2014

Ing. Marco Molina, PhD

Le tre relatività

- Galileiana
 - «Nei sistemi di riferimento inerziali le leggi della meccanica sono le stesse»
- Ristretta
 - « c = costante in tutti i sistemi di riferimento»
- Generale
 - «Le traiettorie orbitali sono geodetiche dello spazio-tempo»

La Meccanica Celeste e l'Astrodinamica

- Newton

- Forza di attrazione **UNIVERSALE**

- $F = G \frac{m \cdot M}{R^2}$

La Meccanica Celeste e l'Astrodinamica

- Newton
 - Forza di attrazione universale
 - $F = G \frac{m \cdot M}{R^2}$
 - Principio di equivalenza: la massa nell'equazione di Newton è la stessa della seconda legge della Dinamica $F = m \times a$

A. Einstein, 1922

- The breakthrough came suddenly one day. I was sitting on a chair in my patent office in Bern. Suddenly a thought struck me: If a man falls freely, he would not feel his weight. I was taken aback. This simple thought experiment made a big impression on me. This led me to the theory of gravity. I continued my thought: A falling man is accelerated. Then what he feels and judges is happening in the accelerated frame of reference. I decided to extend the theory of relativity to the reference frame with acceleration. I felt that in so doing I could solve the problem of gravity at the same time.

10^{-13}

Accuratezza delle misure correnti (a terra) che confermano la validità del principio di equivalenza



Credits: NASA

Esperimenti per la prova del principio di equivalenza (10^{-15})

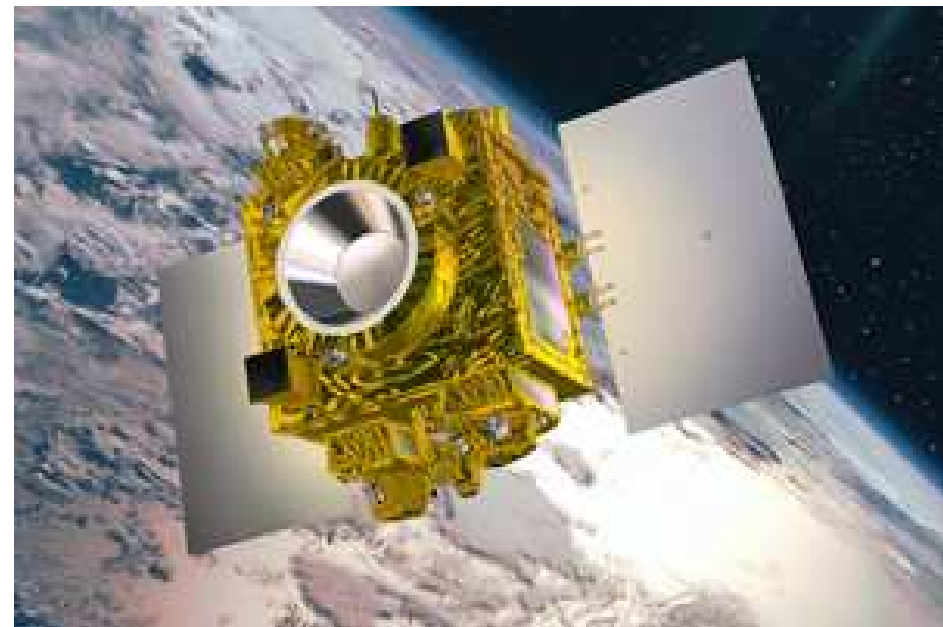
- Pronto per il volo (2015-2016)
 - CNES: MICROSCOPE (Micro-Satellite à trainée Compensée pour l'Observation du Principe d'Equivalence)
- Proposte
 - STEP
 - GG
 - ASTROD
 - STE-Quest

Microscope

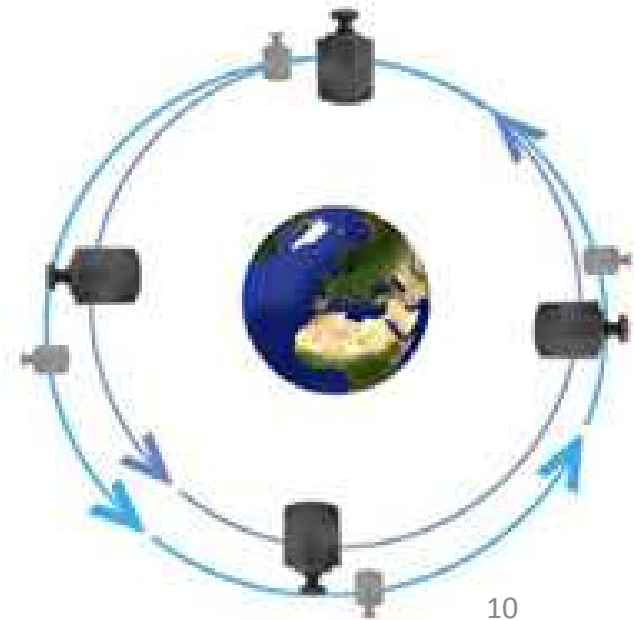


Drag control

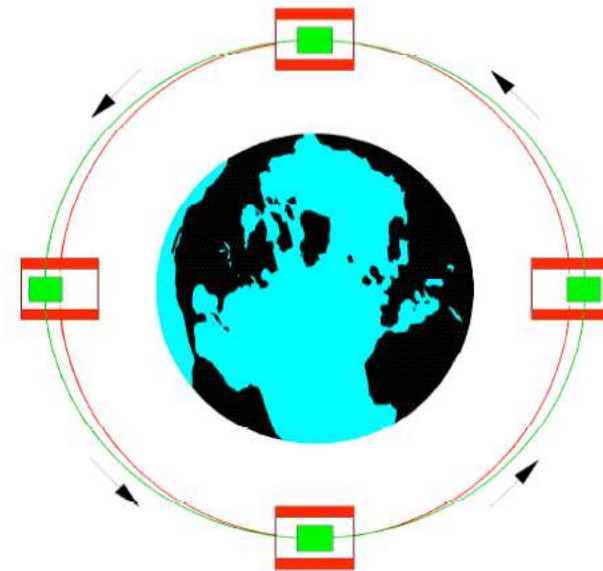
Micropropulsione
1 micro newton



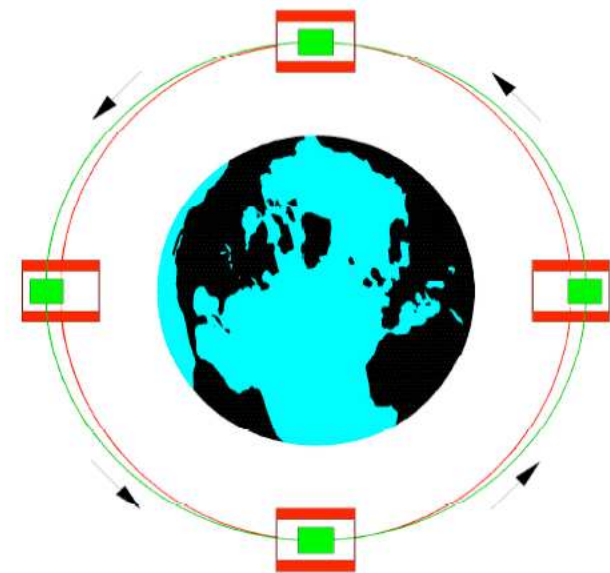
Pt, Ti test masses



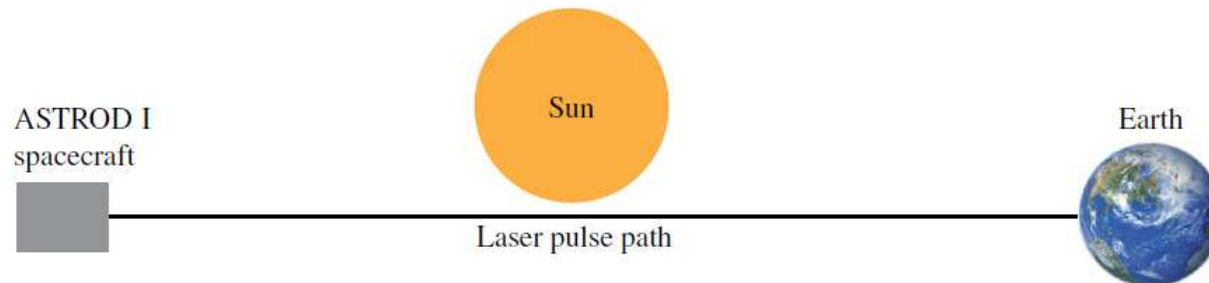
- STEP (1992)= Satellite Test of Equivalence Principle
- ASTROD (1996) (Astrodynamical Space Test of Relativity using Optical Devices)
- GG (1997) = Galileo Galilei



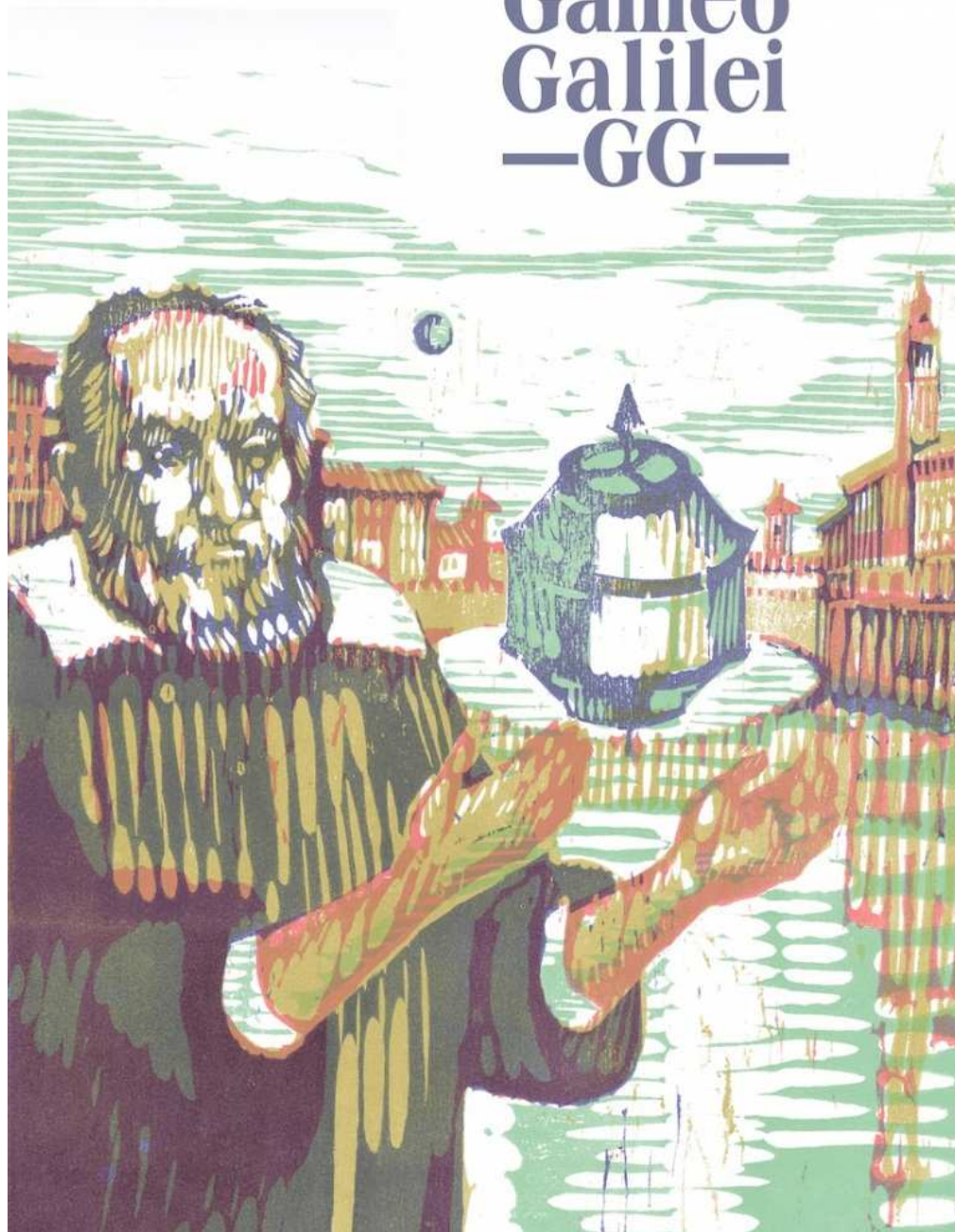
- STEP (1992)= Satellite Test of Equivalence Principle
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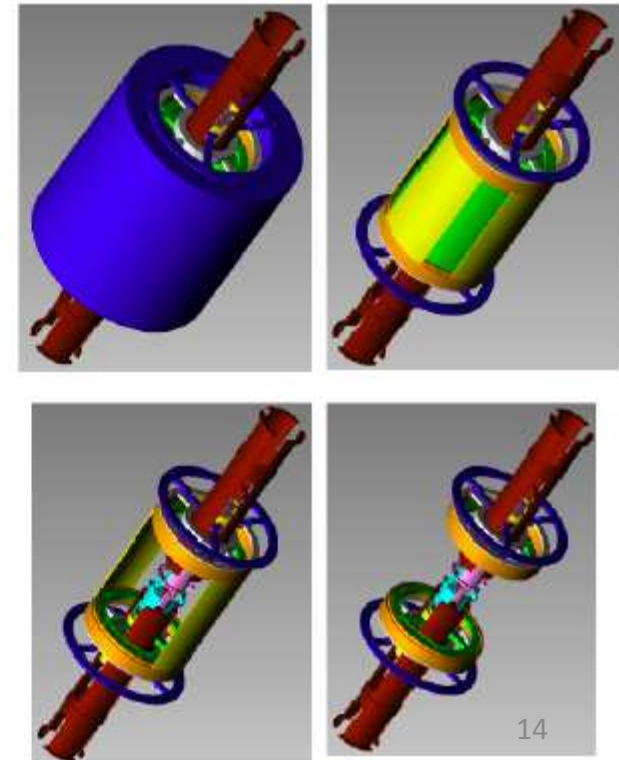
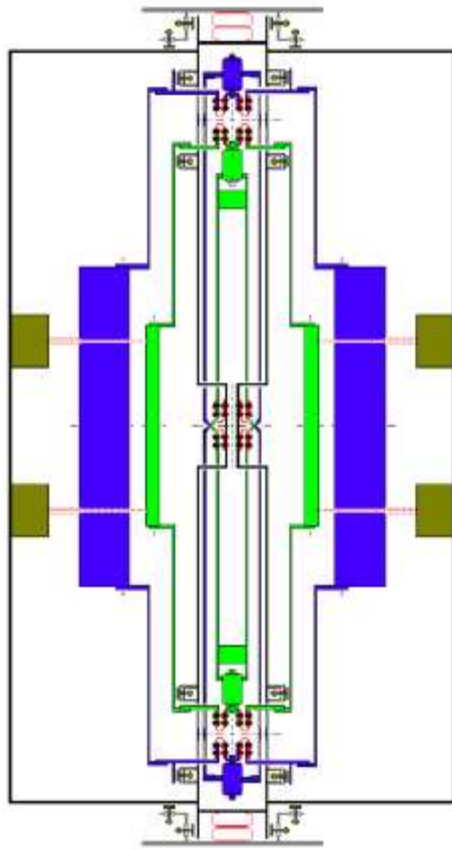
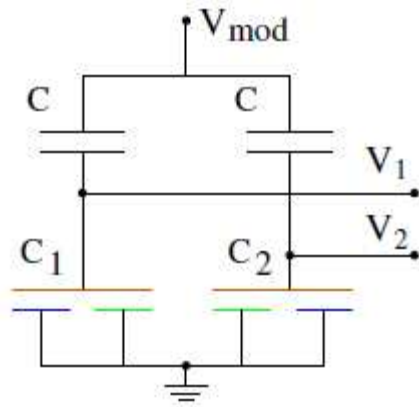
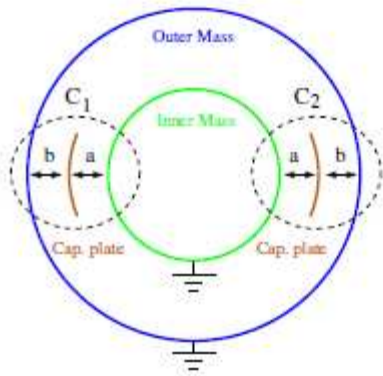
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Galileo Galilei —GG—



Credits: GG Collaboration



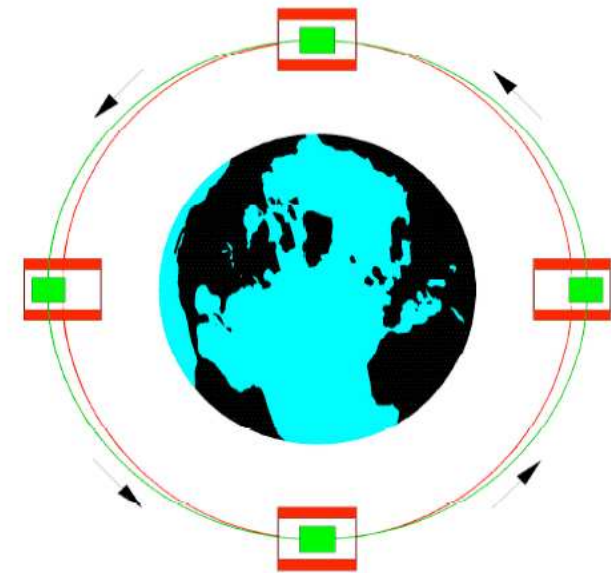
Credits: GG Collaboration

Cosmic Vision

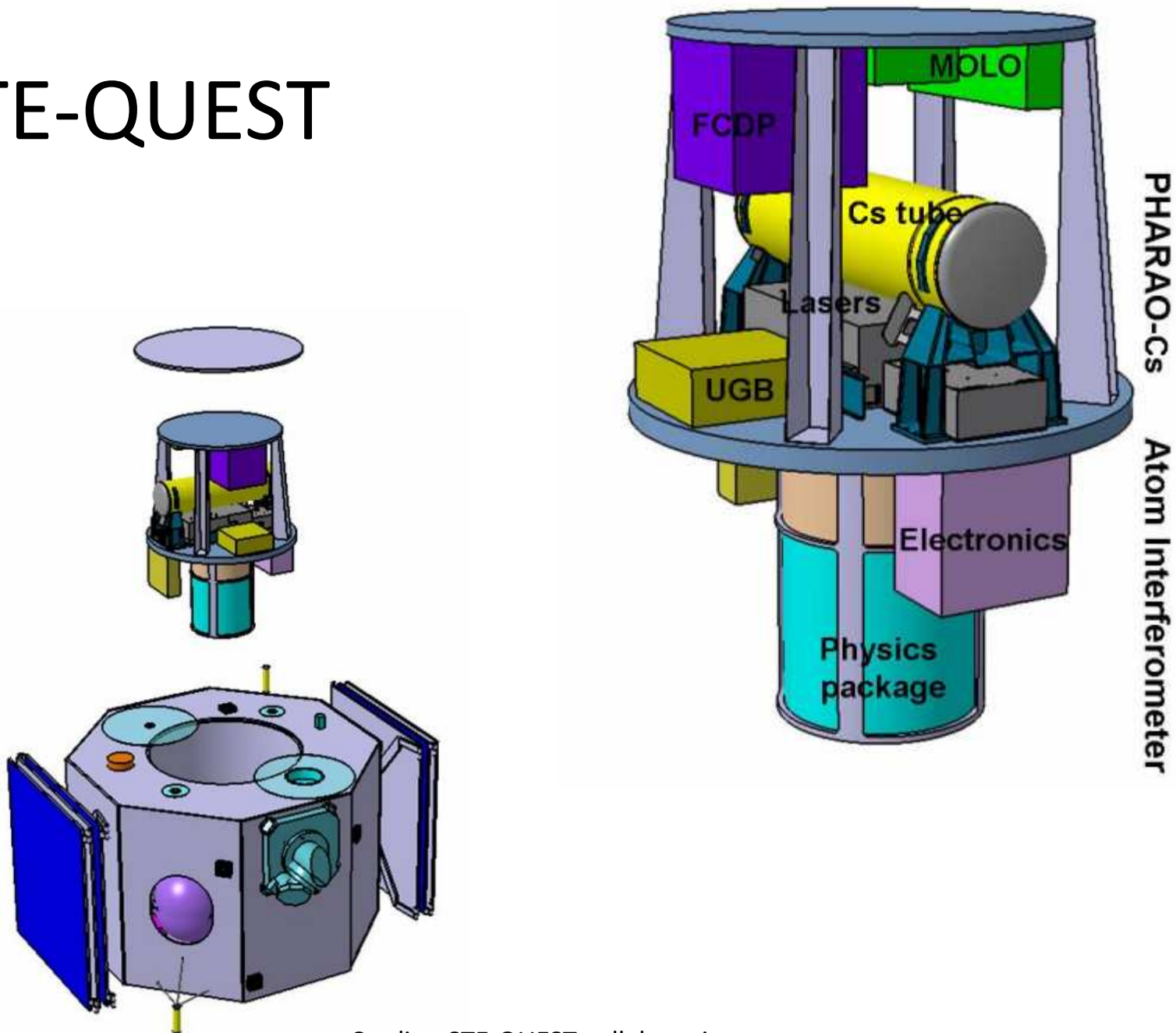
Space Science for Europe 2015-2025



- STEP (1992)= Satellite Test of Equivalence Principle
- ASTROD (1996) (Astrodynamical Space Test of Relativity using Optical Devices)
- GG (1997) = Galileo Galilei
- STE-QUEST (2010 – M3)= Space-Time Explorer and Quantum Equivalence Principle Space Test



STE-QUEST



Credits: STE-QUEST collaboration

Relatività Ristretta

$$\sqrt{1 - (v/c)^2}$$

Velocità nel Sistema Solare

- 7 km/s
- 30 km/s
- 40 km/s
- 200 km/s

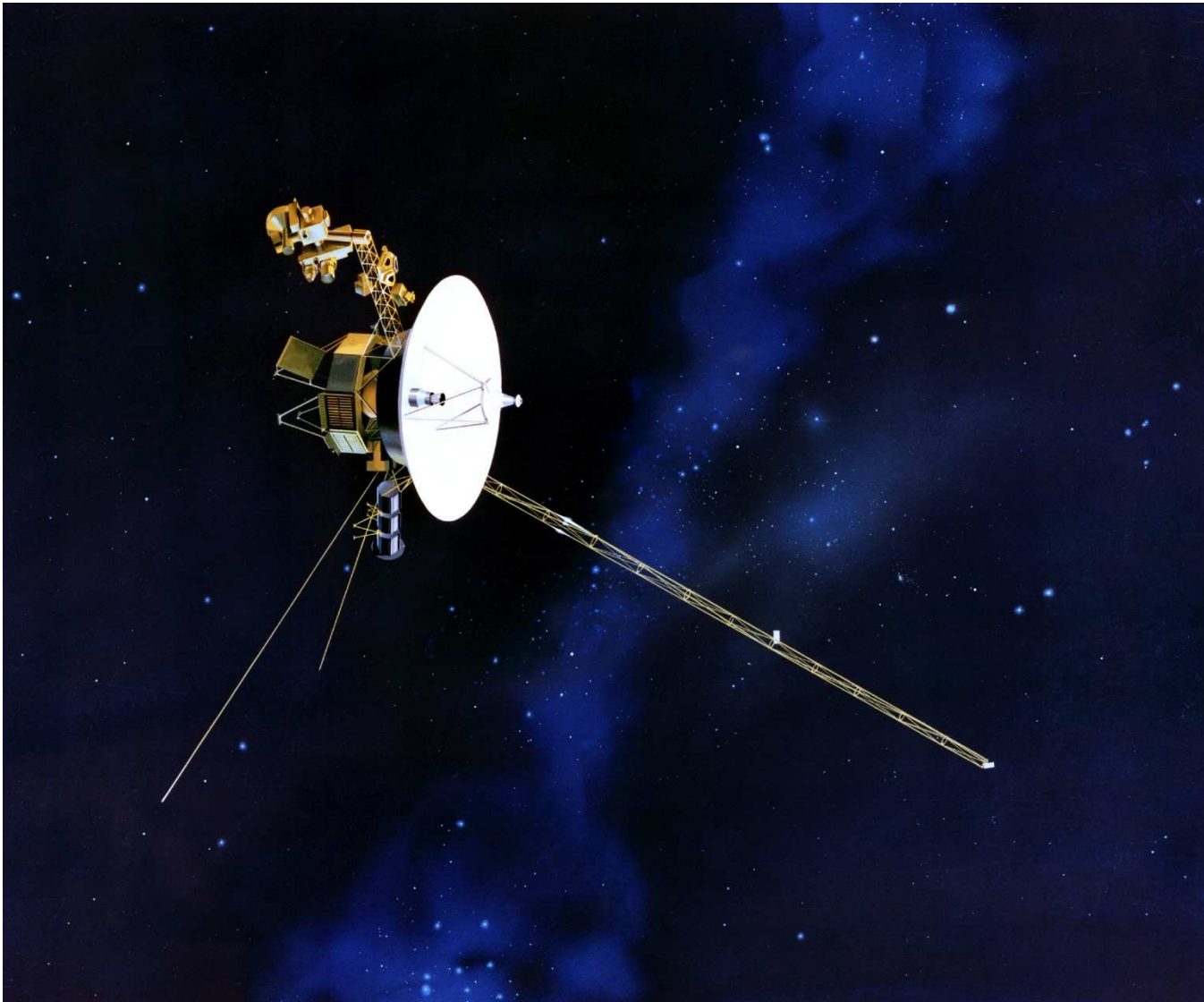
Velocità nel Sistema Solare

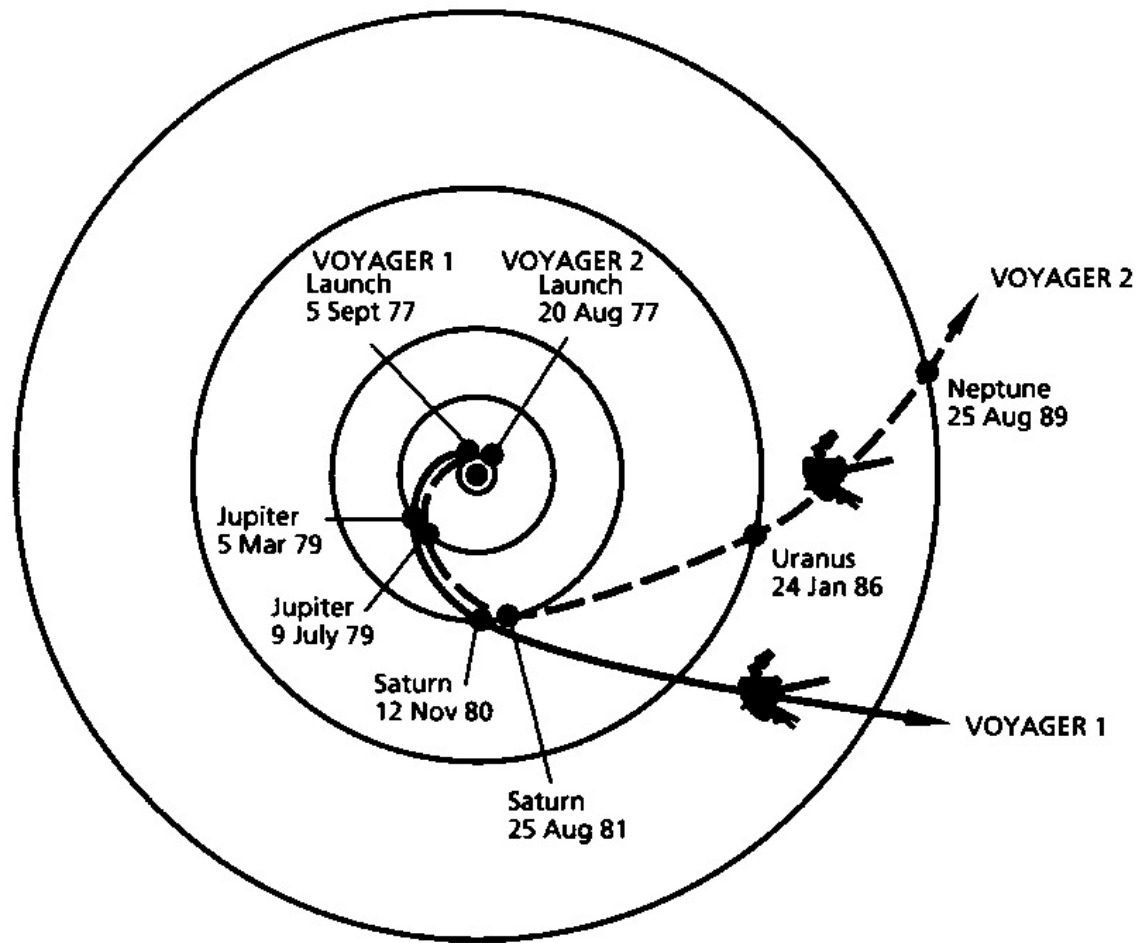
- 7 km/s - Orbita bassa terrestre
- 30 km/s – Moto della Terra attorno al Sole
- 40 km/s – Sonda piu' veloce ora in volo
- 200 km/s – Moto del Sole attorno al centro della Galassia

Velocità nel Sistema Solare

- 7 km/s
- 30 km/s
- 40 km/s
- $200 \text{ km/s} < 0.1\% c \rightarrow 0.99999995 = 0.5 \text{ ppm}$

Voyager (1977) – l'oggetto più lontano costruito dall'uomo





	Voyager 1	Voyager 2
Distance from the Sun (km)	19,330,000,000	15,870,000,000
Distance from the Earth (km)	19,388,000,000	15,855,000,000
Velocity Relative to Sun (km/s)	17	14
Velocity Relative to Earth (km/s)	44	41
Round Trip Light Time (hh:mm:ss)	35:55:42	29:22:32

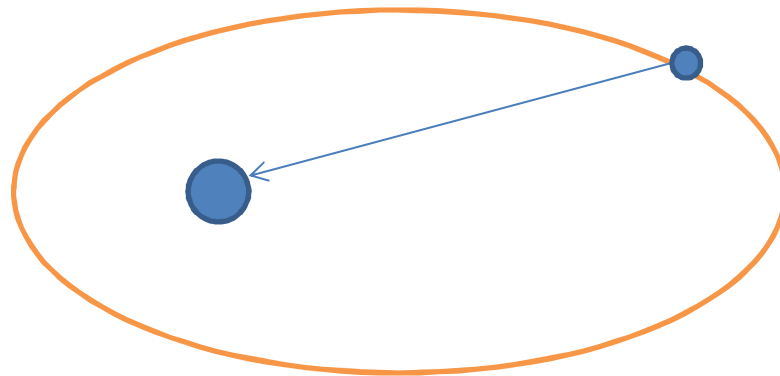
as of 6-Oct-2014

Relatività Generale

Dalle orbite newtoniane alle
Geodetiche dello Spazio-Tempo

Proprietà dei campi di forze centrali

- Forza centrale \rightarrow Conservazione del momento della quantità di moto \rightarrow Seconda legge di Keplero



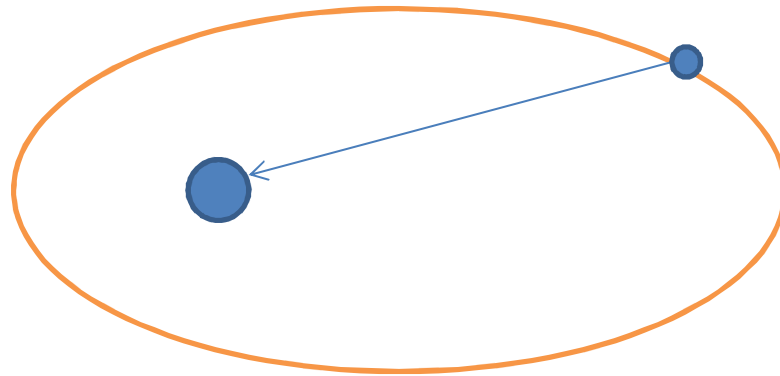
- Orbite planetarie

- Meccanica newtoniana: problema dei due corpi
- Meccanica celeste classica: problema degli N-corpi
- Meccanica celeste relativistica

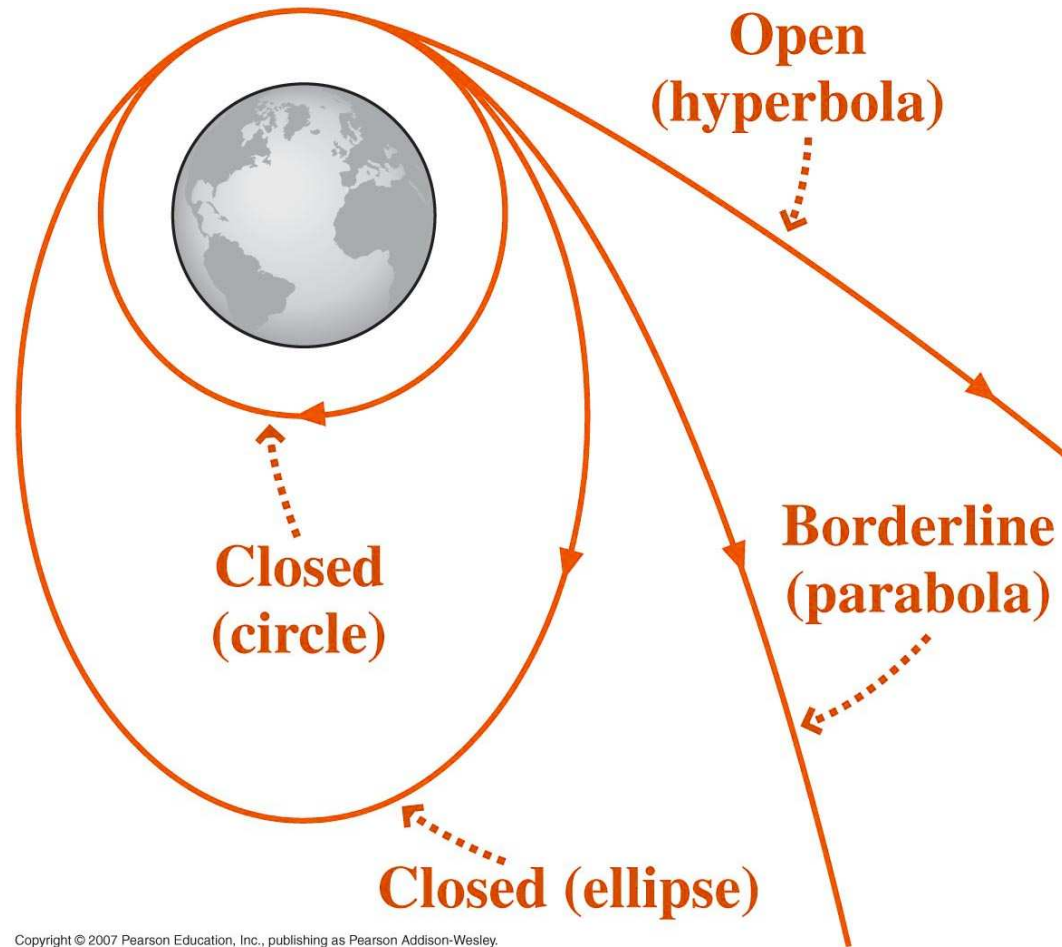
Proprietà dei campi di forze centrali

- Forza centrale \rightarrow Conservazione del momento della quantità di moto \rightarrow Seconda legge di Keplero
- Forza centrale $\neq -\frac{K}{R^2} \rightarrow$ Orbite aperte
- Forza centrale $\neq -KR \rightarrow$ Orbite aperte

FORZA ATTRAZIONE $-\frac{K}{R^2}$



FORZA ATTRAZIONE $-\frac{K}{R^2}$



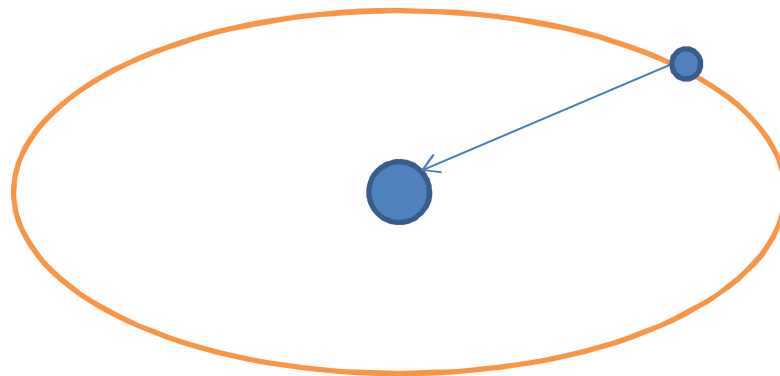
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FORZA ATTRAZIONE – KR

- MOLLA LINEARE IDEALE

$$- F = - kx$$

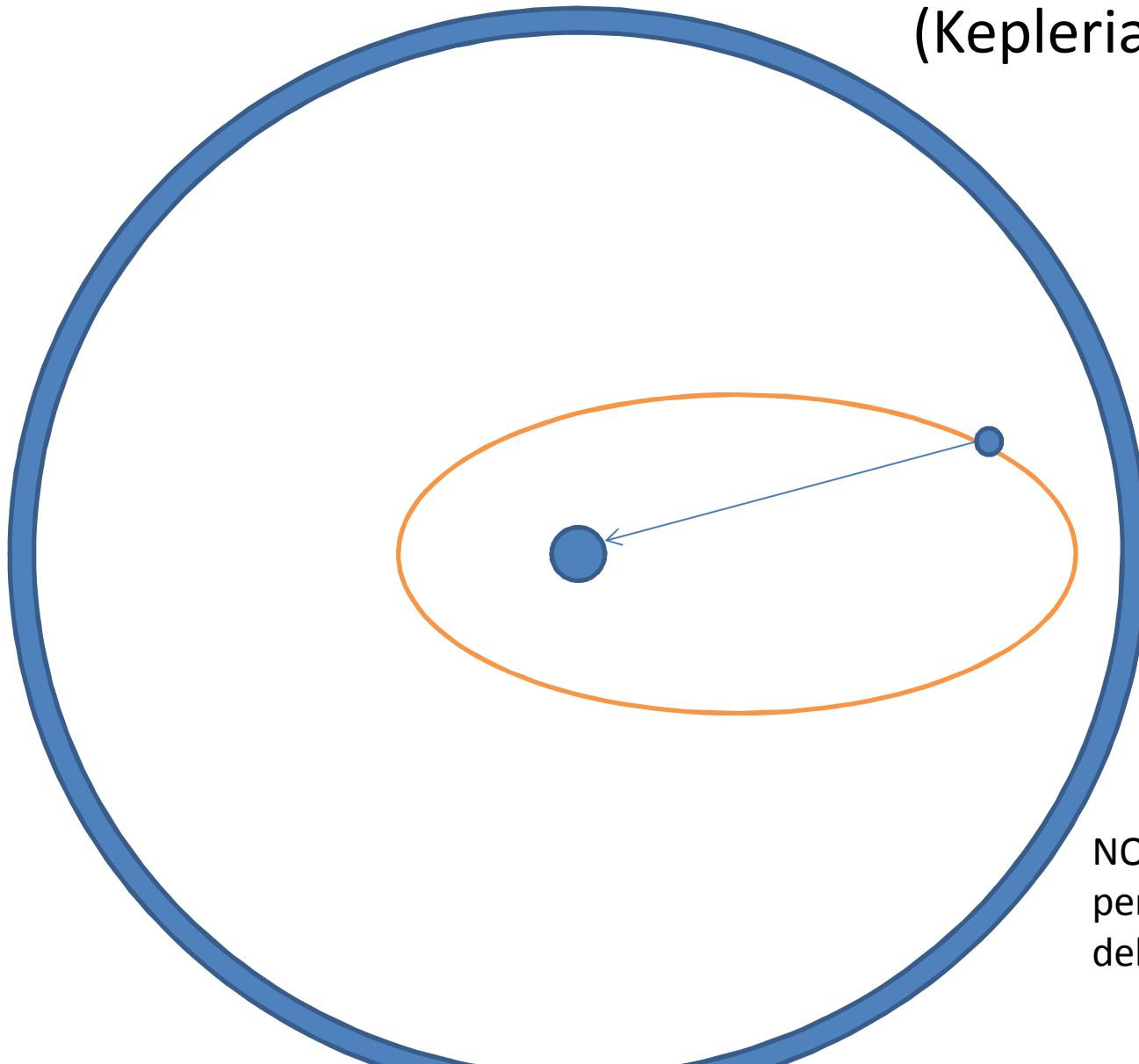
FORZA ATTRAZIONE – KR



- NOTARE l'orbita centrata nel CENTRO dell'ellisse (non nel suo fuoco!)

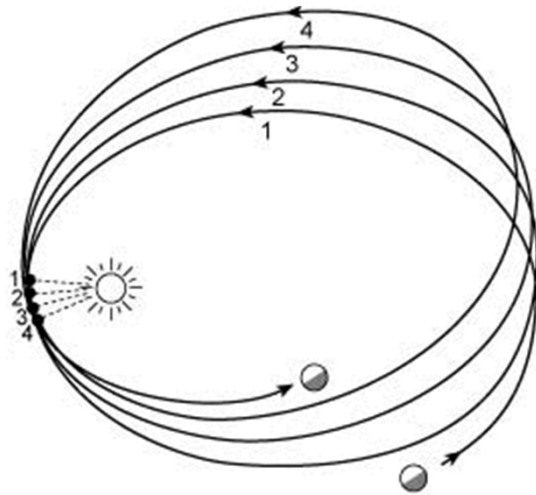
Orbite aperte

(Kepleriane perturbate)



NOTARE l'orbita non
perturbata centrata nel FUOCO
dell'ellisse

Orbite aperte



COMPTE RENDU
DES SÉANCES
DE L'ACADÉMIE DES SCIENCES.



SÉANCE DU LUNDI 12 SEPTEMBRE 1859.

PRÉSIDENCE DE M. DE SENARMONT.



MÉMOIRES ET COMMUNICATIONS
DES MEMBRES ET DES CORRESPONDANTS DE L'ACADÉMIE.

CHIMIE ORGANIQUE. — *Note de M. Biot sur la formation artificielle de l'acide tartrique par M. Liebig.*

ASTRONOMIE. — *Lettre de M. LE VERRIER à M. Faye sur la théorie de Mercure et sur le mouvement du périhélie de cette planète.*

« Vous n'avez peut-être pas oublié combien, dans mes études sur les mouvements de notre système planétaire, j'ai rencontré de difficultés à faire concorder d'une manière complète la théorie avec les observations. Cet accord, disait Bessel il y a trente ans, est toujours affirmé, mais sans qu'on l'ait jusqu'ici vérifié d'une manière assez sérieuse.

» Les écarts reconnus dans le mouvement d'Uranus ont été expliqués par la découverte de Neptune.

» Mais, ce qui est remarquable, c'est qu'il a suffi d'augmenter de 38 secondes le mouvement séculaire du périhélie pour représenter toutes les observations des passages à moins d'une seconde près, et même la plupart d'entre elles à moins d'une demi-seconde. Ce résultat si net, qui donne immédiatement à toutes les comparaisons une exactitude supérieure à celle qu'on a obtenue jusqu'ici dans les théories astronomiques, montre clairement que l'accroissement du mouvement du périhélie de Mercure est indispensable, et qu'à cette condition les Tables de Mercure et du Soleil jouissent de toute la précision désirable.

j'ai rencontré de difficultés à faire concorder d'une manière complète la théorie avec les observations. Cet

- **Le Verrier 1865** Avanzamento del perielio di Mercurio «Non riesco a far tornare la teoria con le osservazioni»

– TEORIA	531 arcsec/secolo
– OSSERVAZIONI	569 arcsec/secolo

» Mais, ce qui est remarquable, c'est qu'il a suffi d'augmenter de 38 secondes le mouvement séculaire du périhélie pour représenter toutes les observations des passages à moins d'une seconde près, et même la plupart d'entre elles à moins d'une demi-seconde.

38 arcsec/secolo

Le Verrier 1865

+ 5025 arcsec/secolo	precessione degli equinozi
+ 531 arcsec/secolo	N corpi
+ 0.0254arcsec/secolo	J2 Sole
= 5557 arcsec/secolo TOTALE (TEORIA)	

5595 arcsec/secolo OSSERVATA (misura del 1865)

2015

+ 5025 arcsec/secolo	precessione degli equinozi
+ 531 arcsec/secolo	N corpi
+ 0.0254arcsec/secolo	J2 Sole

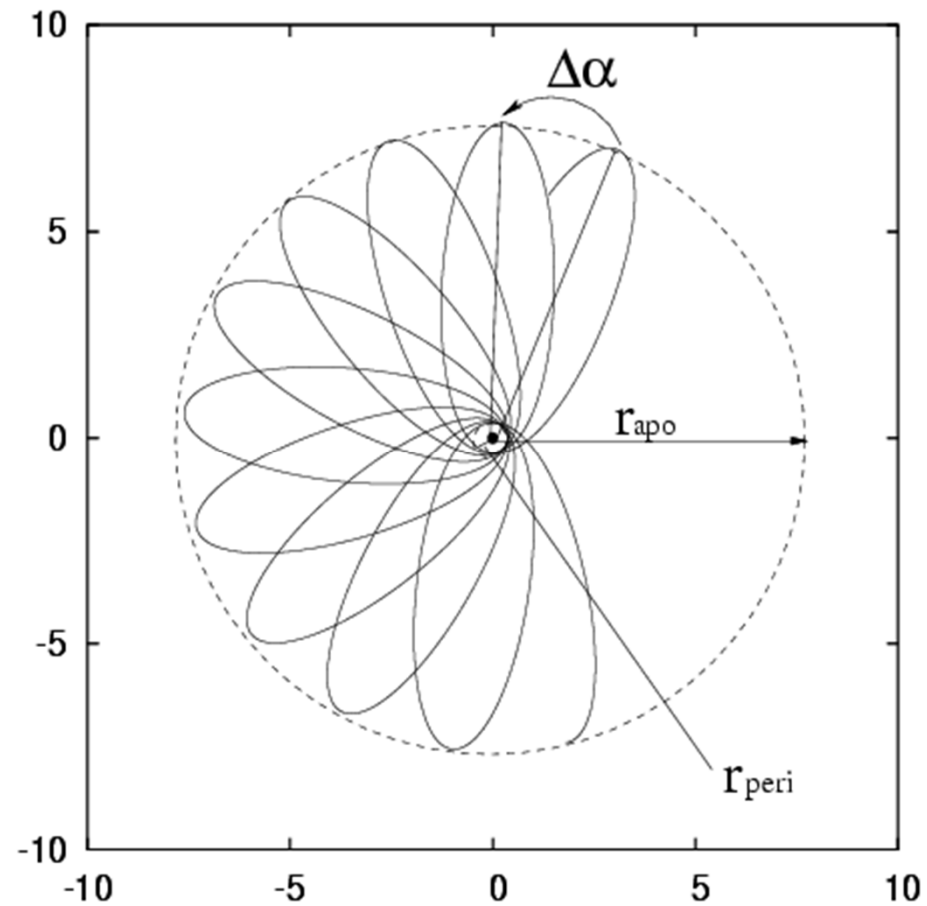
+ 43 arcsec/secolo

R.G.

= 5599 arcsec/secolo TOTALE (TEORIA, RELATIVITA')

5599.7 arcsec/secolo OSSERVATA nel 2015

Poligasteroide («Rosetta» di Mercurio)

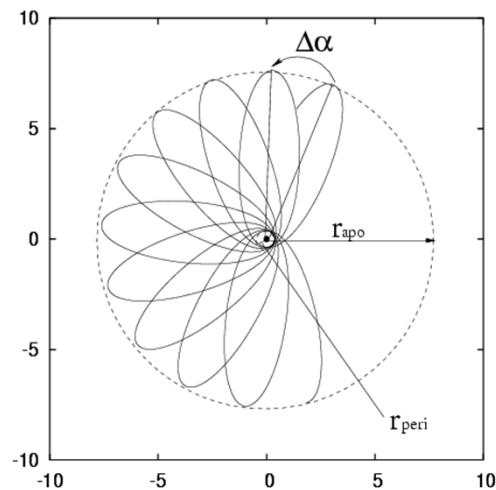


Avanzamento del perielio

$$\delta\varphi \approx \frac{6\pi G(M + m)}{c^2 A (1 - e^2)}$$

arc seconds per century

	GR	observed
Mercury	43.0	43.1 ± 0.5
Venus	8.6	8.4 ± 4.8
Earth	3.8	5.0 ± 1.2
Icarus (asteroid)	10.1	9.8 ± 0.8



GRAVITY PROBE-B GPB

- 750 Milioni \$
 - 31 anni di R&D
 - 10 anni di preparazione al volo
 - 1.5 anni in volo
 - 5 anni di analisi dati
- **Risultati finali di GP-B sono stati annunciati alla NASA HQ a Washington DC il 4 Maggio 2011**

CONFERME di GP-B (1)

- Previsione teorica di Einstein sull'effetto geodetico (curvatura dello Spazio-Tempo) con errore dello 0.28%

magnetismo

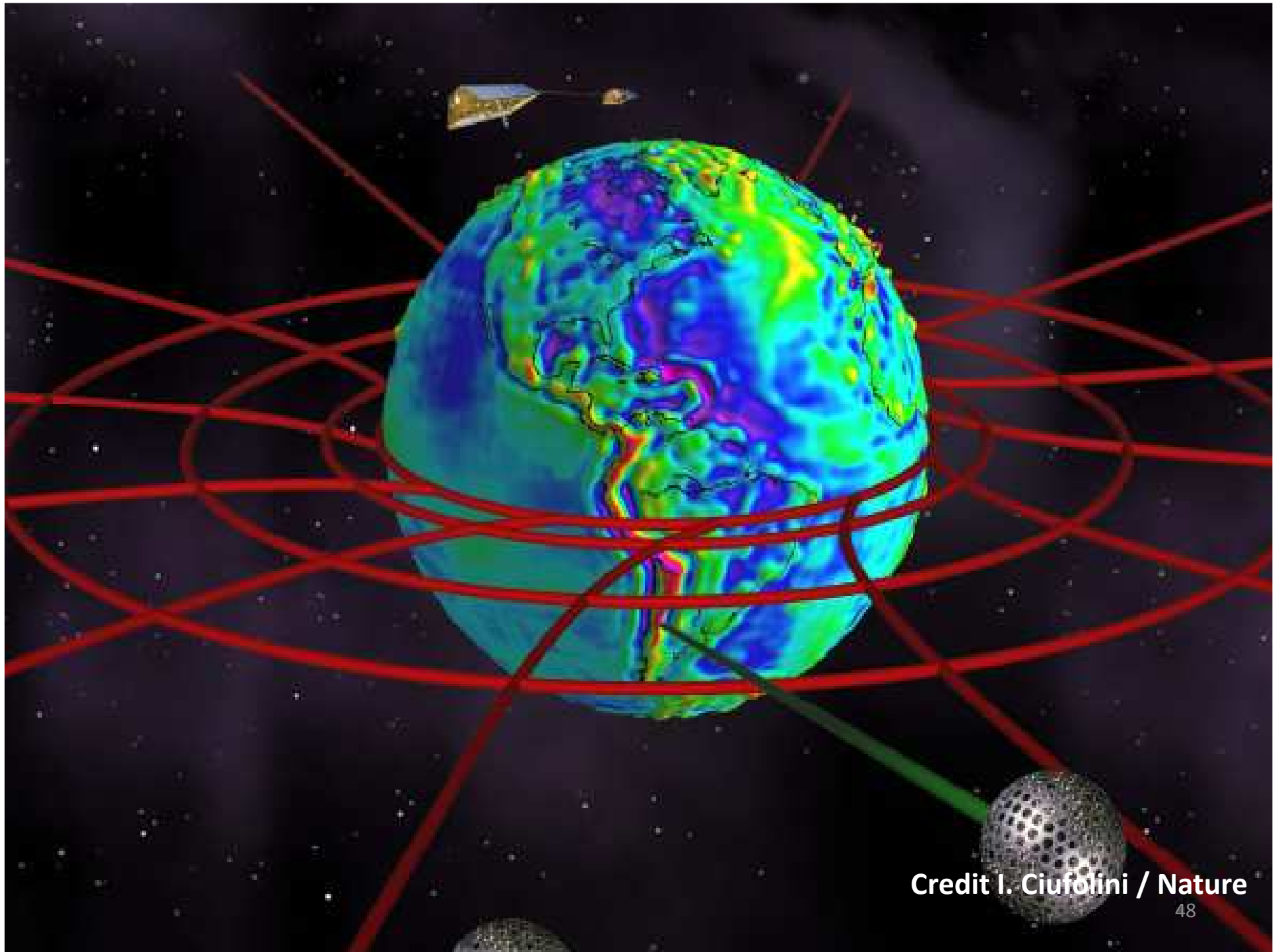
- Corrente genera → campo magnetico → deflette ago magnetizzato

Gravitomagnetismo

- Corrente genera \rightarrow campo magnetico \rightarrow deflette ago magnetizzato
- Massa rotante genera \rightarrow campo gravitomagnetico
 \rightarrow deflette asse di un giroscopio

- Il **frame-dragging rotazionale** (o effetto Lense-Thirring) secondo il principio della relatività generale e le teorie consimili, è avvertibile in prossimità di oggetti massivi rotanti.

$$\Omega_{\text{LT}} = -\frac{2}{5} \frac{Gm\omega}{c^2 R} \cos\theta.$$



Credit I. Ciufolini / Nature

CONFERME di GP-B (2)

- Previsione teorica di Einstein sull'effetto geodetico (curvatura dello Spazio-Tempo) con errore dello 0.28%
- Previsione teorica di “frame-dragging” (rotazione dello Spazio-Tempo) con errore del 19%



Credit NASA / GPB

IM Pegasi



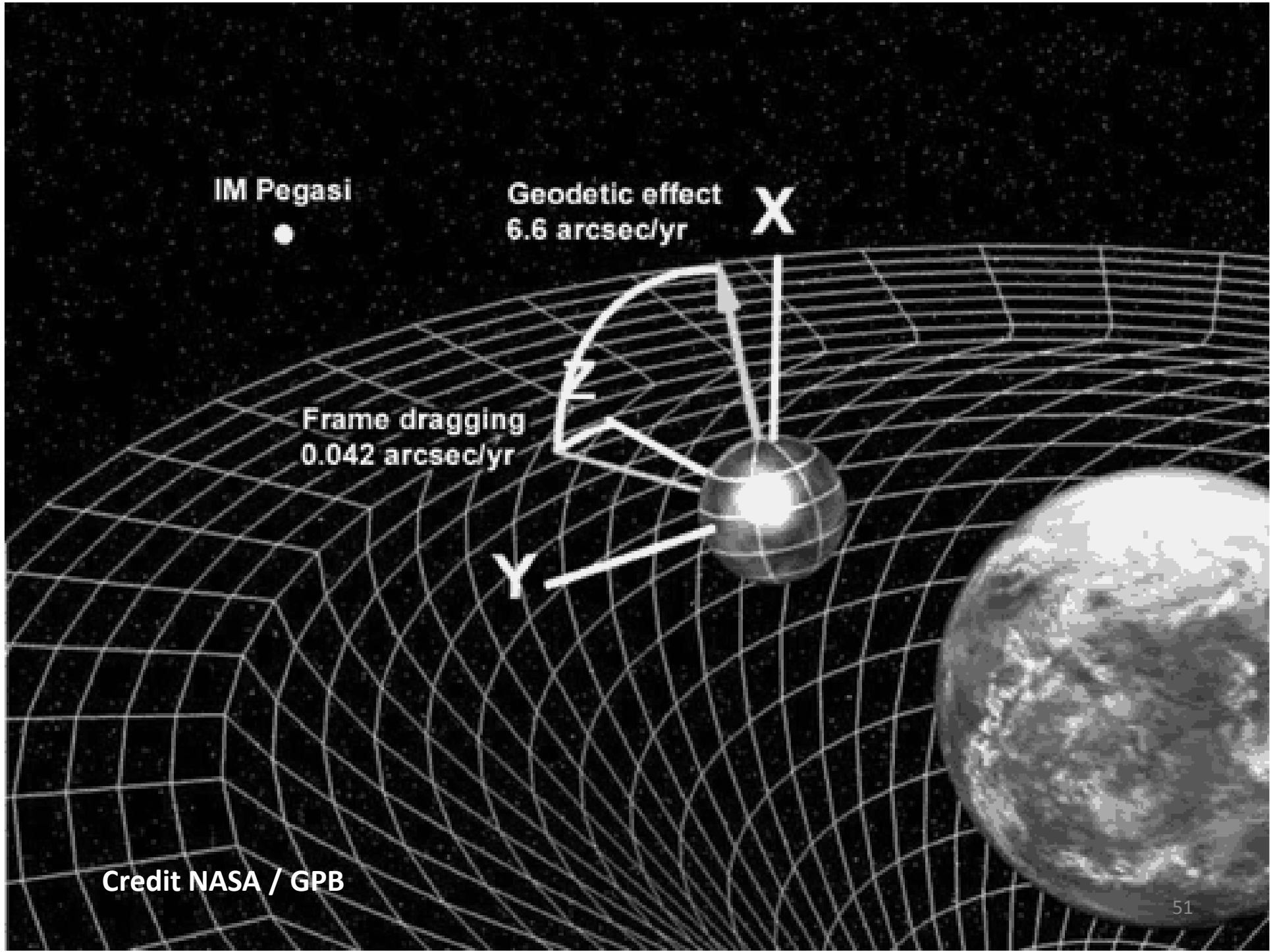
Geodetic effect
6.6 arcsec/yr

X

Frame dragging
0.042 arcsec/yr

Y

Credit NASA / GPB



Guide Star
IM Pegasi
(HR 8703)

Frame-dragging Precession

39 milliarcseconds/year
(0.000011 degrees/year)

642 kilometers
(~400 miles)

Geodetic Precession

6,606 milliarcseconds/year
(0.0018 degrees/year)

$$\Omega = \frac{3GM}{2c^2 R^3} (R \times v) + \frac{GI}{c^2 R^3} \left[\frac{3R}{R^2} (\omega \cdot R) - \omega \right]$$

Geodetic Precession

Frame-dragging Precession

Credit NASA / GPB

GP-B results

- MISURE

- Geodetico = -6601.8 ± 18.3 milliarcsec/anno
Frame-dragging drift rate = -37.2 ± 7.2
milliarcsec/anno

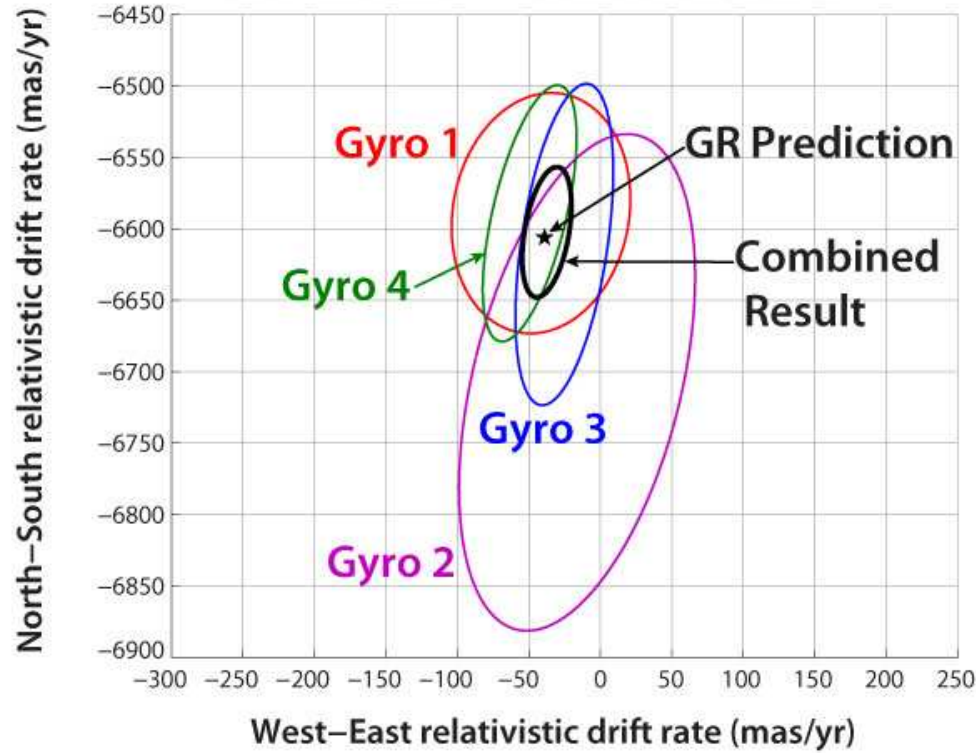
- TEORIA RELATIVITA' GENERALE

- Geodetico = -6606.1 milliarcsec/anno
- Frame-dragging drift rate = -39.2 milliarcsec/anno



The Gravity Probe B EXPERIMENT

Final Results — December 2010



North-South & West-East Relativistic Drift Rates, 95% Confidence

Four-Gyro Weighted Average Results

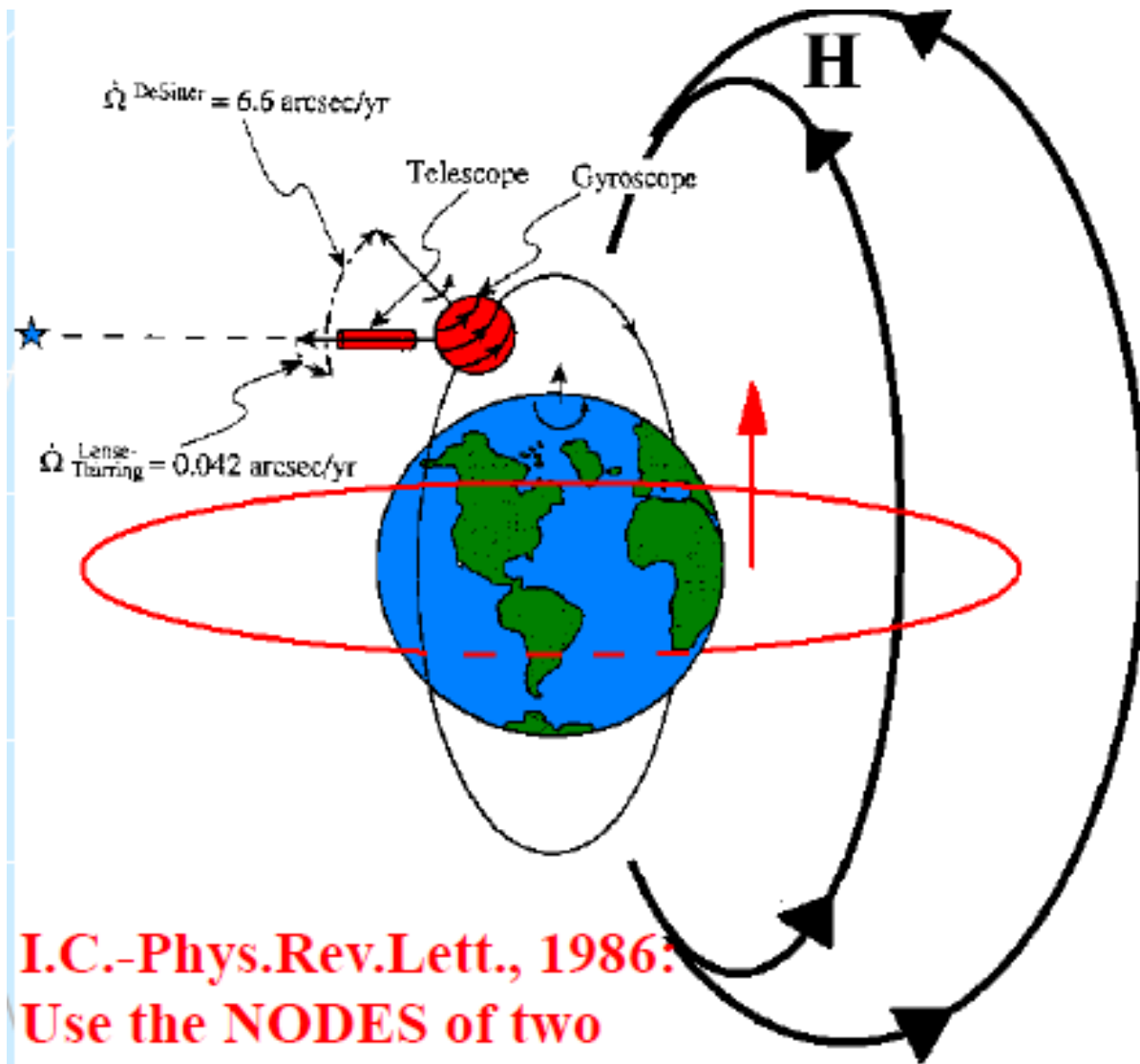
Geodetic (N-S): -6602 ± 18.4 mas/yr; Statistical Margin of Error: 0.28%

Frame-Dragging (W-E): -37.4 ± 7.3 mas/yr; Statistical Margin of Error: 19%

Einstein Predictions

Geodetic (N-S): -6606 mas/yr Frame-Dragging (W-E): -39 mas/yr

Credit NASA / GPB



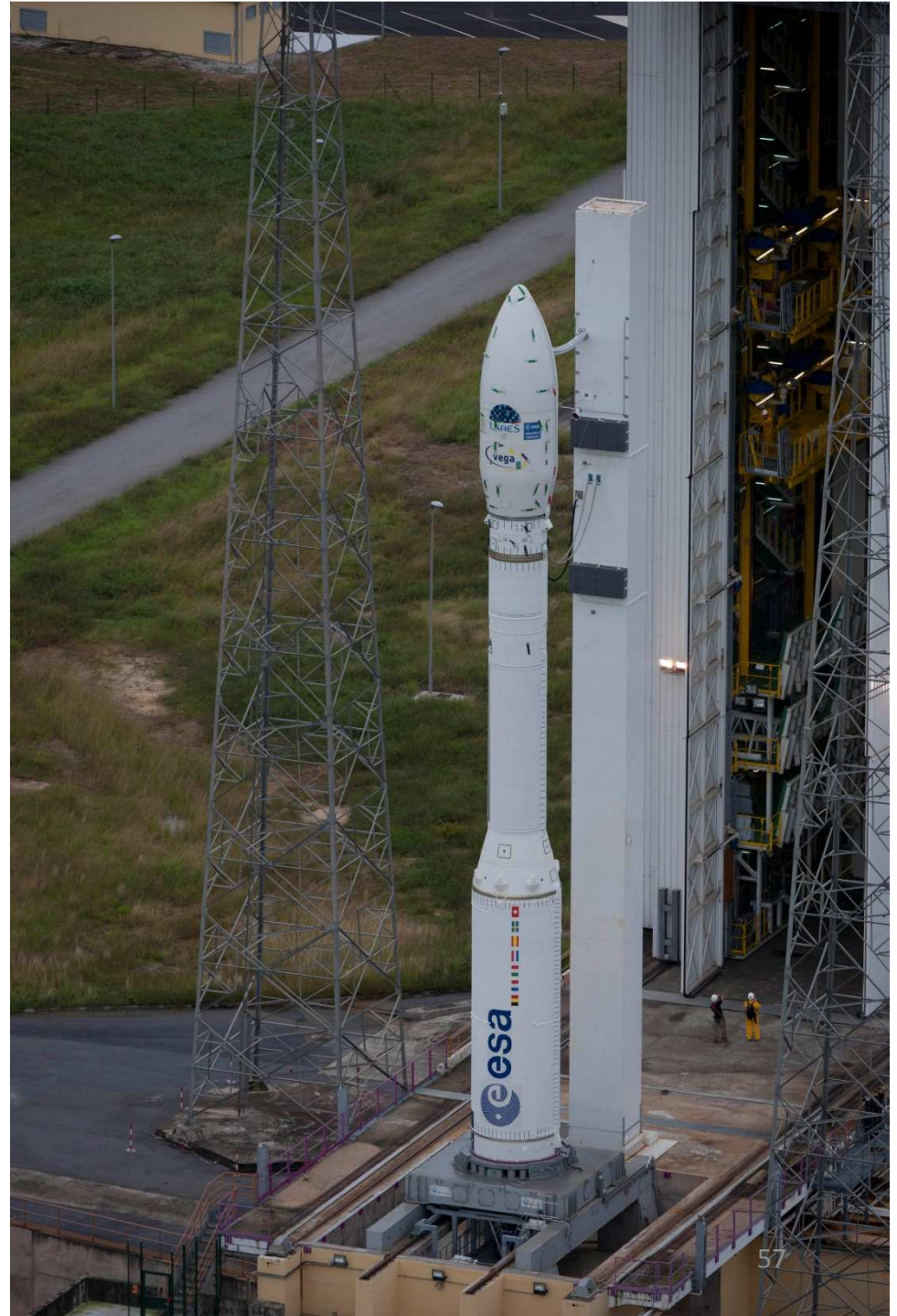
**I.C.-Phys.Rev.Lett., 1986:
 Use the NODES of two
 LAGEOS satellites; the orbital plane of these
 satellites is a huge gyroscope affected by
 frame-dragging. This is called the Lense-Thirring
 effect**

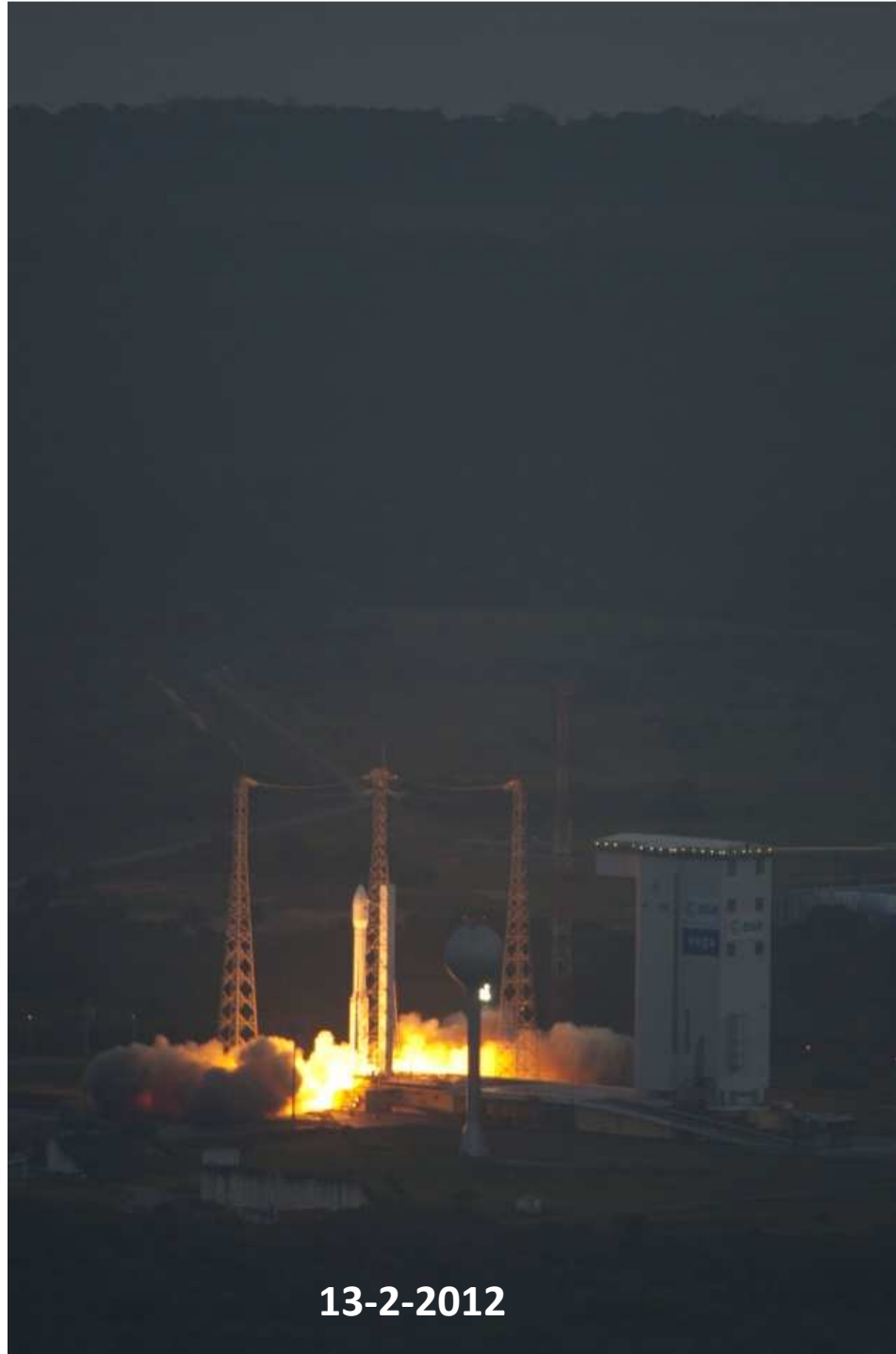
2007





Credits:ESA

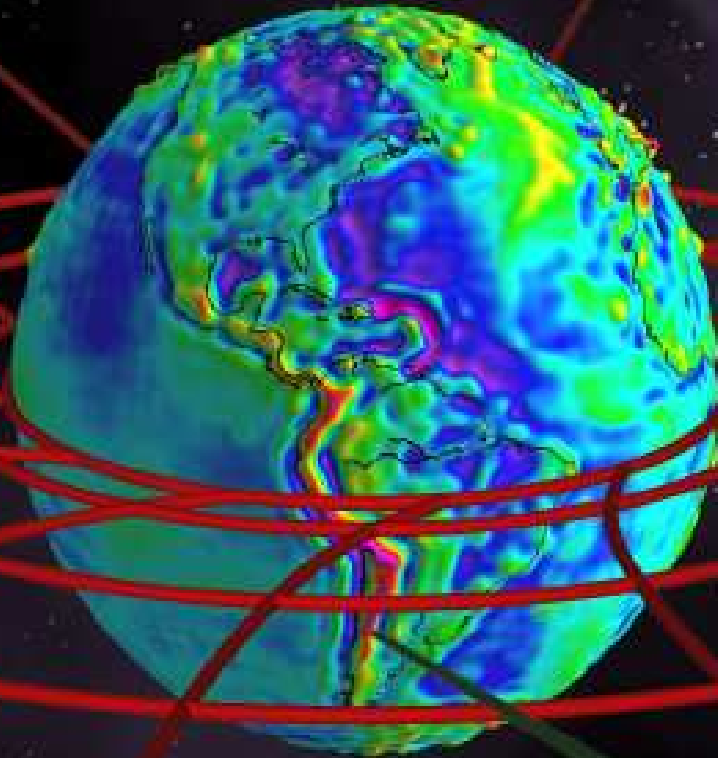


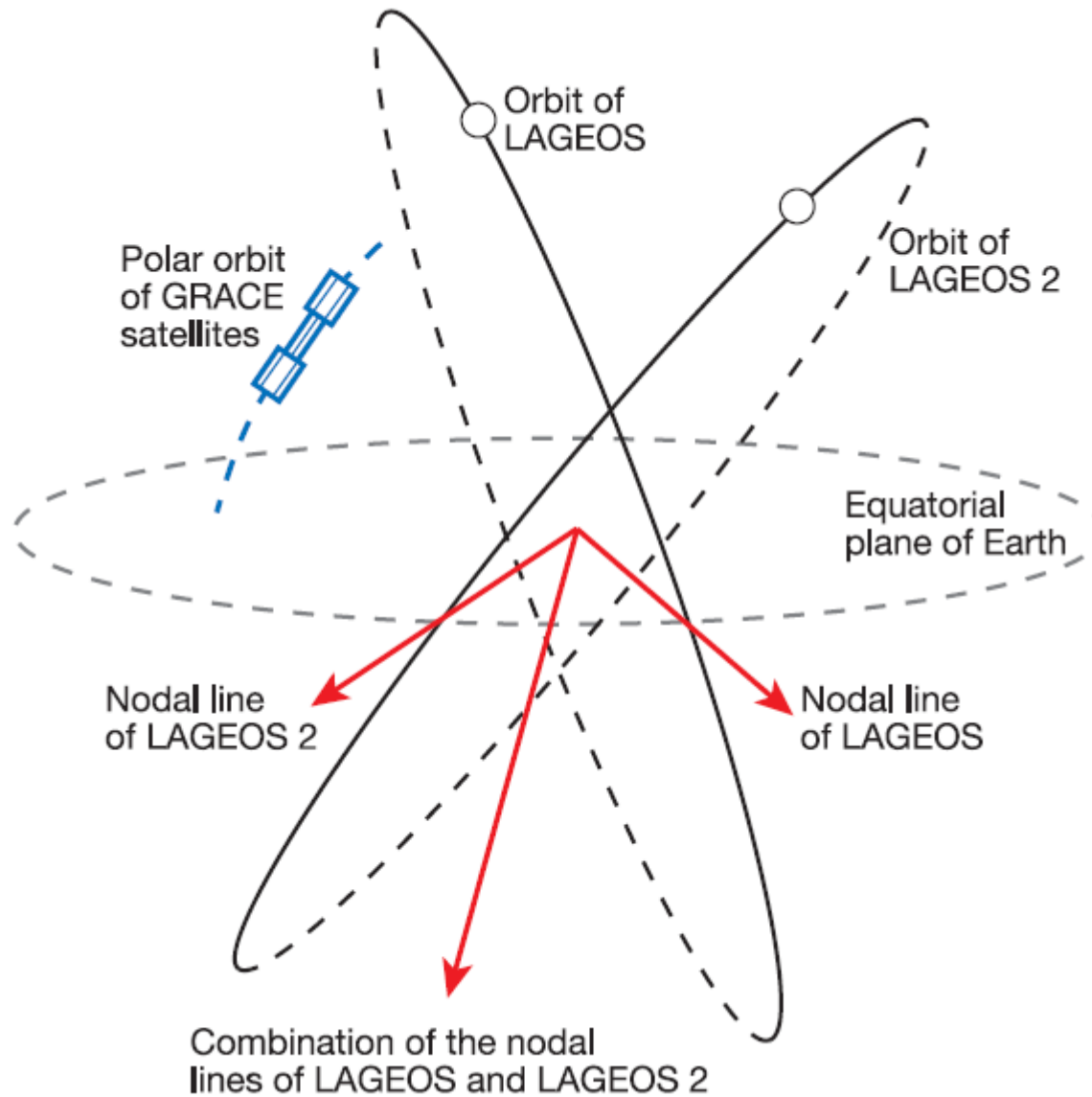


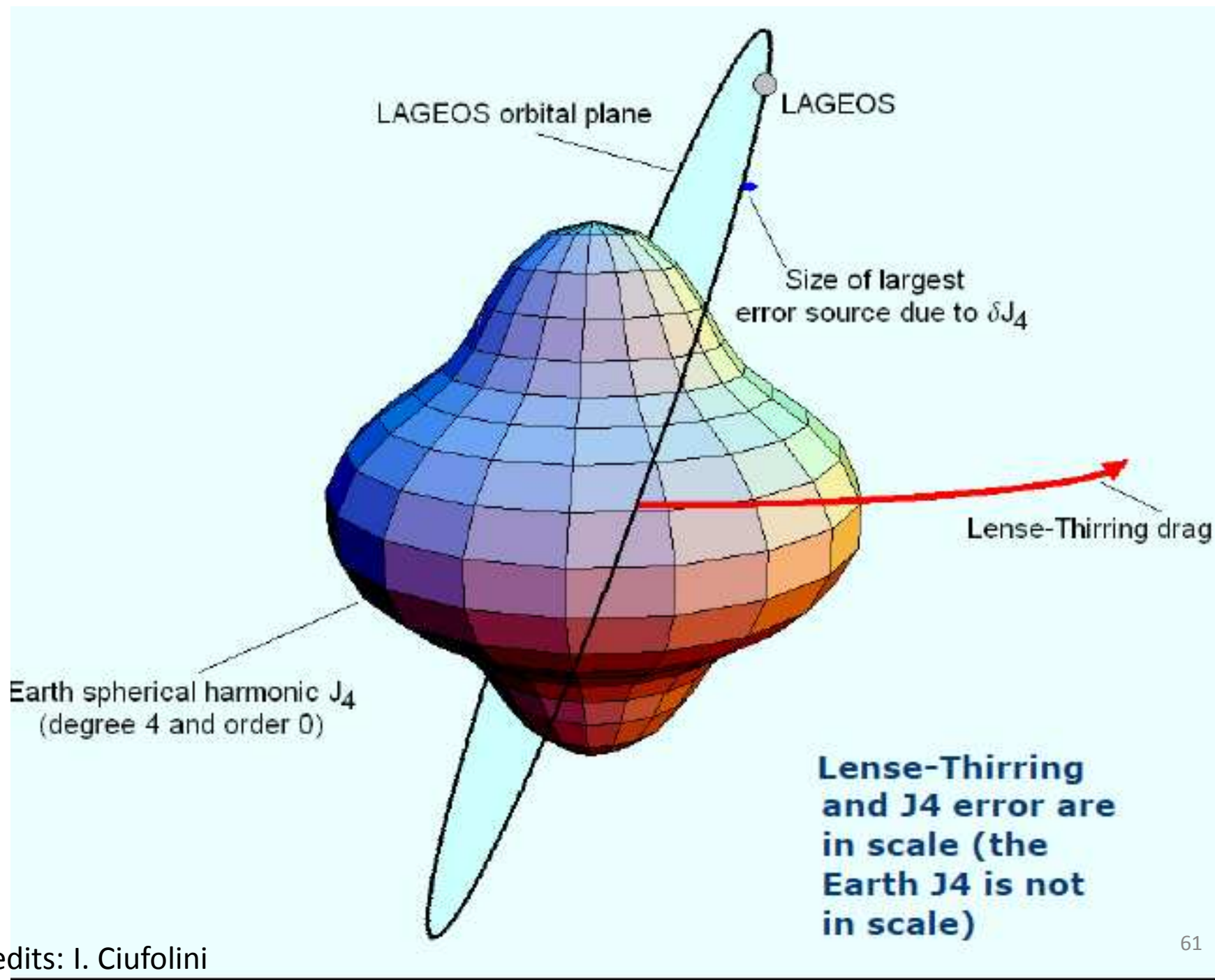
Credits:ESA

13-2-2012

58







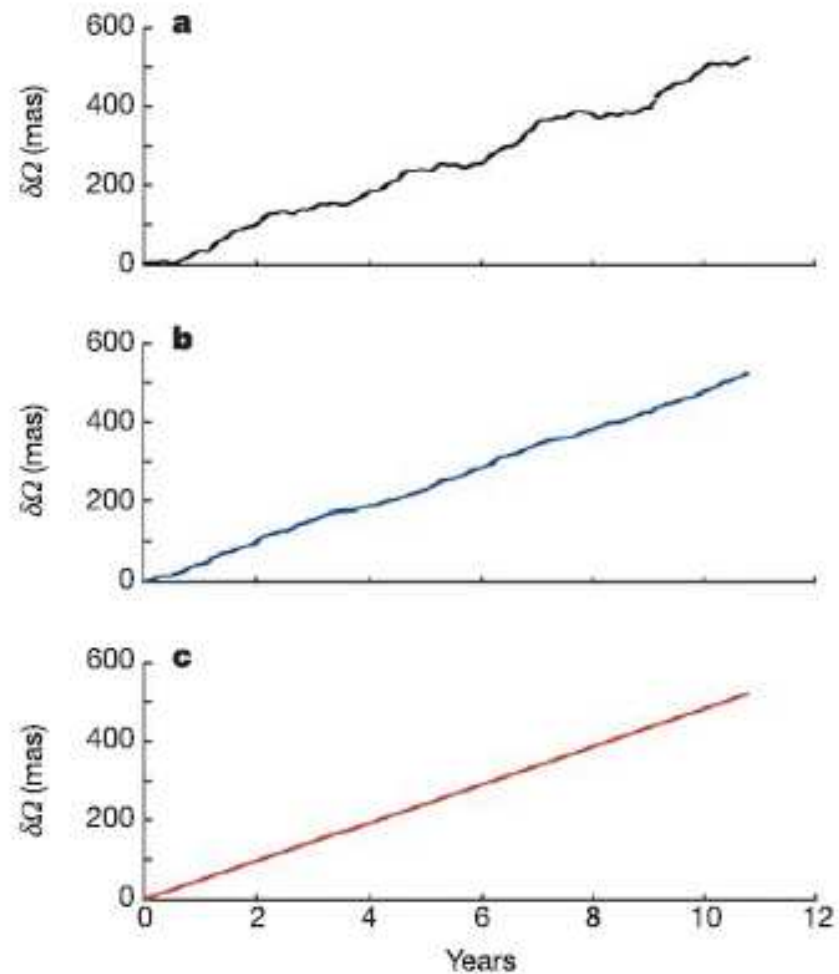


Figure 2 Observed orbital residuals of the LAGEOS satellites. The residual nodal longitudes of the LAGEOS satellites, $\delta\Omega$, were combined according to equation (1). In black (a) is the raw, observed, residual nodal longitude of the LAGEOS satellites without removal of any signal, whereas in blue (b) is the observed residual nodal longitude after removal of six periodic signals. The best-fit line (13-parameter fit) through these observed residuals has a slope of 47.9 mas yr^{-1} . In red (c) is the theoretical Lense–Thirring prediction of Einstein’s general relativity for the combination (equation (1)) of the nodal longitudes of the LAGEOS satellites; its slope is 48.2 mas yr^{-1} .

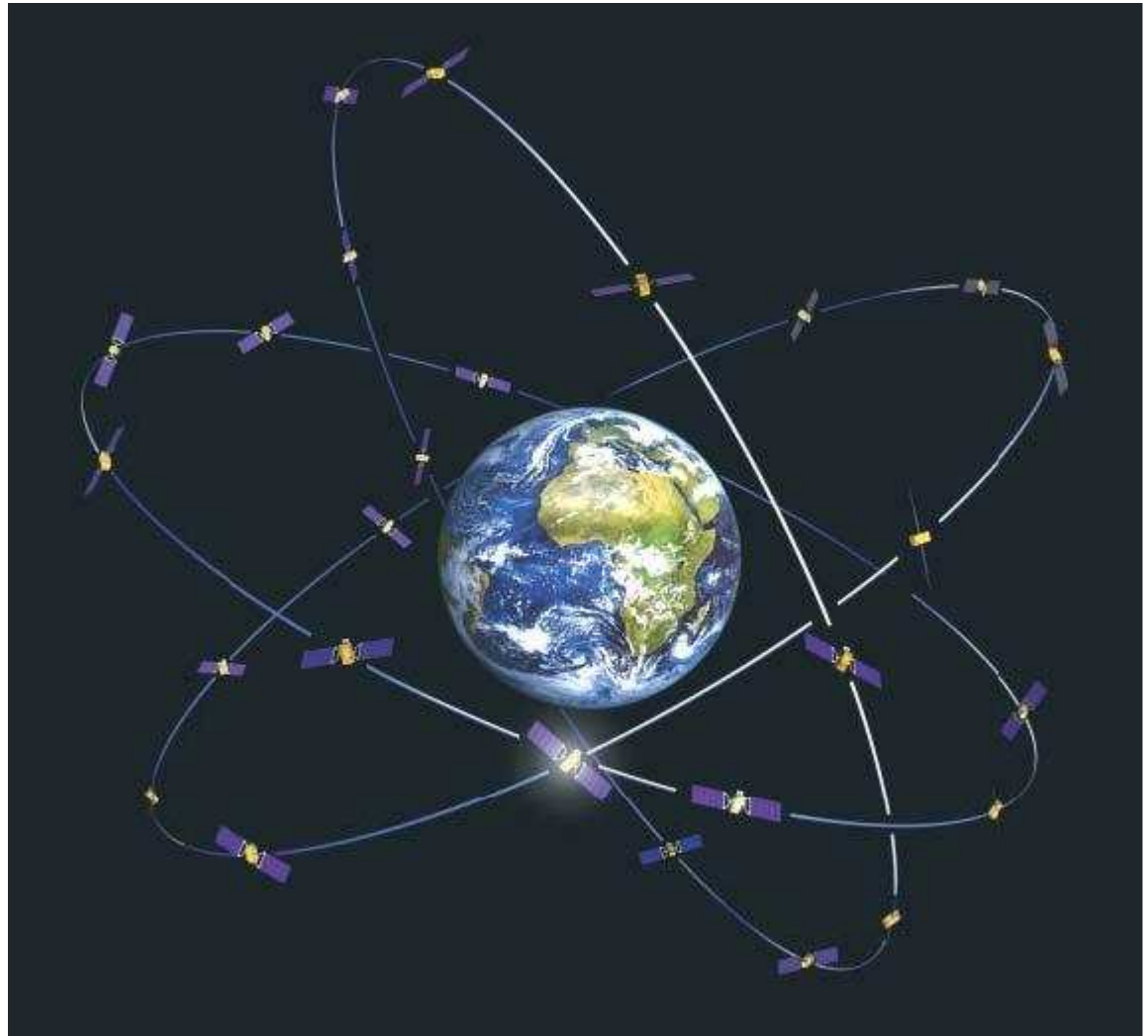
GPS

- 6 Piani orbitali x
4 satelliti = GPS



GALILEO

- 6 Piani orbitali x
4 satelliti = GPS
- 3 Piani orbitali x
9 satelliti =
GALILEO



GPS

- Accuratezza richiesta = 20-30 nanosecondi.

GALILEO

- Accuratezza richiesta = 20-30 nanosecondi.



Space-borne Passive Hydrogen MASER (PHM)

On-board Master Clock for the Galileo Navigation System
Physic Package (PP) developed and manufactured by Spectra Time SA

MAIN FEATURES

- Magnetron-type microwave cavity
- Bulk getter pumped hydrogen vacuum system
- Solid-state hydrogen storage
- Automatic Cavity Tuning system (ACT)
- Stability guaranteed also on ground

OTHER CHARACTERISTICS

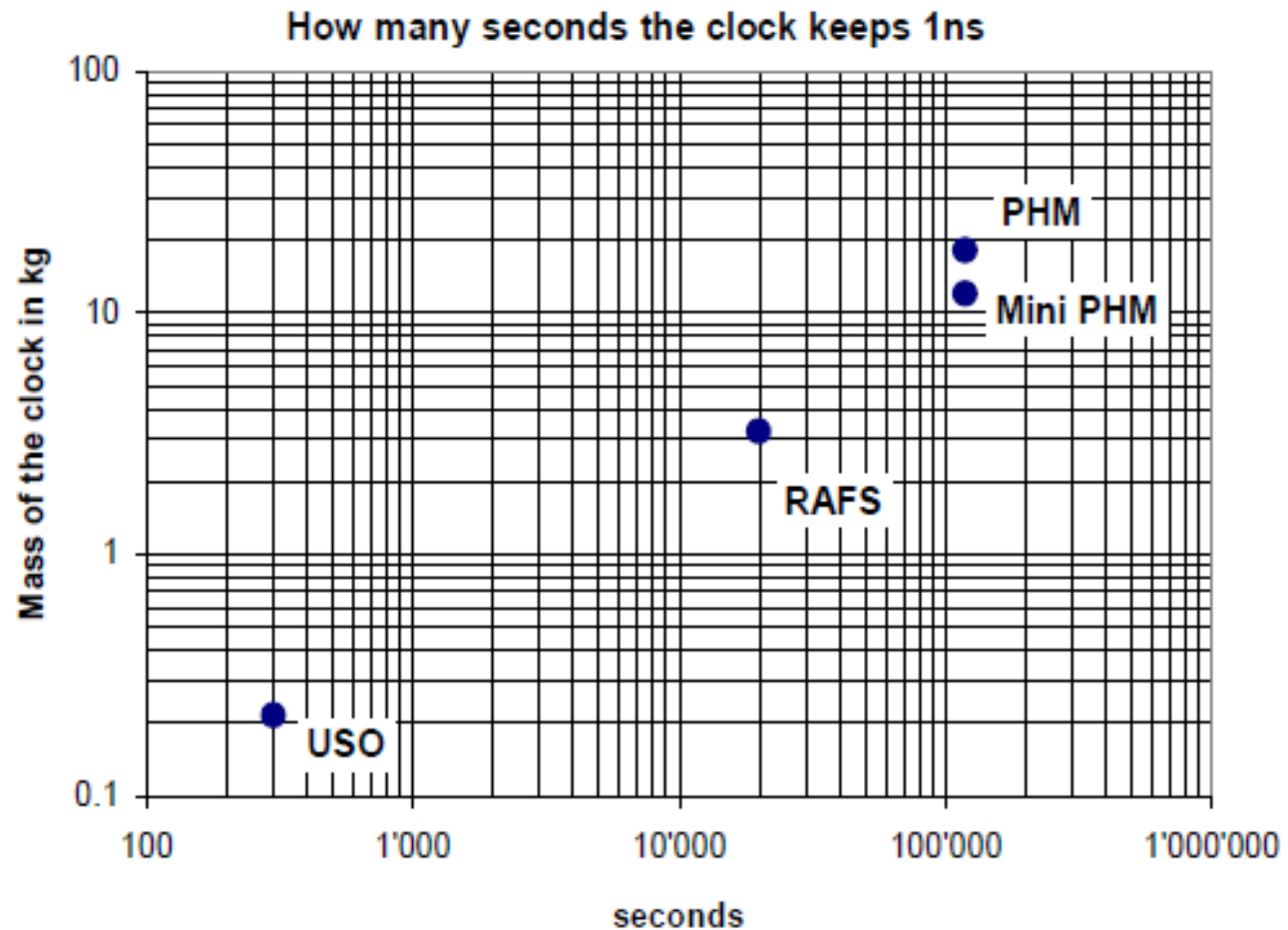
Satellite	Galileo Constellation
Launch Date	From 2008
Output Frequency	10.0028574 MHz ($f_H/142$)
Frequency Drift/Year	$\leq 3 \times 10^{-12}$
Allan deviation, $1s < \tau < 10^4 s$	$\leq 1 \times 10^{-12} \times \tau^{-1/2}$
Temperature sensitivity	$\pm 1 \times 10^{-14} / ^\circ C$
Voltage sensitivity	$\leq 1 \times 10^{-15} / V$
Magnetic Sensitivity	$\leq 2 \times 10^{-14} / Gauss$
Dimensions	210 x 537 x 250 mm ³
Weight	$\leq 18 Kg$
Main Bus Voltage	50 V \pm 1 V
Power consumption	$\leq 70 W$
Qualification Temperature Range	-15 to +20 $^\circ C$
Lifetime (MEO Orbit)	>12 years



The PHM on board the GIOVE B payload

Why use PHM for navigation?

- 1ns equals 30cm of precision
- Minimum 8hrs of operation without relying on ground station



Stimiamo gli effetti relativistici su un orologio atomico di Galileo

Effetto della Relatività Ristretta

- $\sqrt{1 - \frac{v^2}{c^2}} \approx 1 - \frac{1}{2} \frac{v^2}{c^2}$

- $v = 4000 \text{ m/s}$

- $\sqrt{1 - \frac{v^2}{c^2}} \approx 1 - \frac{1}{2} \frac{v^2}{c^2} = 1 - 8.35 \times 10^{-11}$

GPS/Galileo

- Accuratezza richiesta = 20-30 nanosecondi.
- Relatività ristretta = $-8.35E-11 \times 86400 =$
-7 microsecondi/giorno
7.000 nanosecondi/giorno
- Dopo 2 ore circa = errore!

Effetto Doppler relativistico

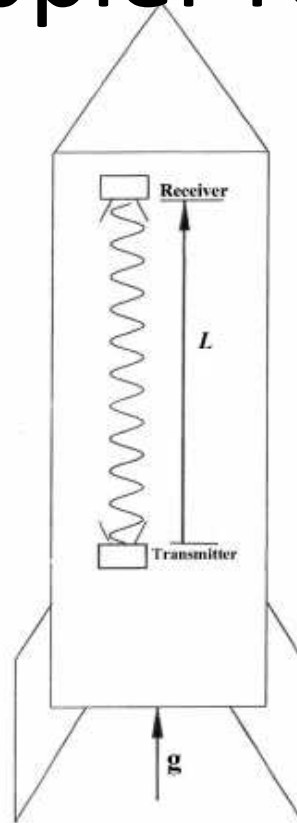


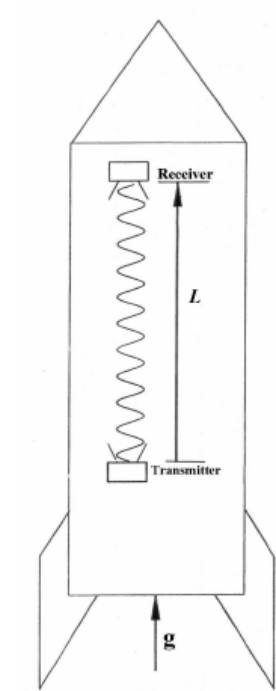
FIG. 6: A signal rising up in an accelerating rocket will be redshifted; the Principle of Equivalence then leads to a prediction of the gravitational frequency shift.

- $\frac{\Delta f}{f} = -\frac{v}{c}$

- $v = gt = g \frac{L}{c}$

Effetto Doppler

Moto accelerato



- $\frac{\Delta f}{f} = -\frac{v}{c}$

Effetto Doppler

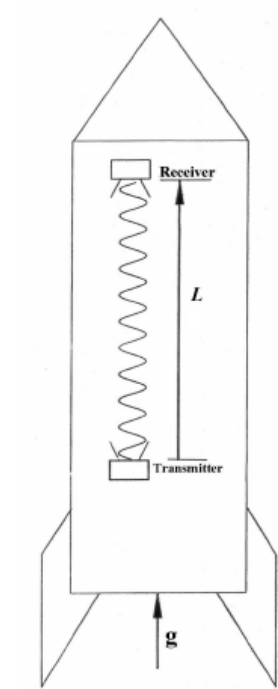
- $v = gt = g \frac{L}{c}$

Moto accelerato

- $\frac{\Delta f}{f} = g \frac{L}{c^2}$

- $gL = -\Delta\varphi$

Potenziale gravitazionale



©M. Molina 2014

$$\Phi = -\frac{GM}{r}$$

Effetto della Relatività Generale

$$\frac{\Delta f}{f} = -\frac{\Delta\varphi}{c^2}$$

$$\frac{\Delta f}{f} = \frac{\varphi - \varphi_{REF}}{c^2} = -\frac{GM}{c^2 r} - \frac{GM}{c^2 a} \approx 5.28 \times 10^{-10}$$

- $5.28 \times 10^{-10} \times 86400 = +45$ microsecondi /giorno

GPS/Galileo

- Accuratezza richiesta = 20-30 nanosecondi.
- Relatività ristretta = -7 microsecondi/giorno

GPS/Galileo

- Accuratezza richiesta = 20-30 nanosecondi.
- Relatività speciale = -7 microsecondi/giorno
- Relatività Generale = +45 microsecondi/giorno

GPS

- Accuratezza richiesta = 20-30 nanosecondi.
- Relatività speciale = -7 microsecondi/giorno
- Relatività Generale = +45 microsecondi/giorno
- TOTALE = +38 microsecondi/giorno
- TOTALE = 38.000 nanosecondi/giorno
- Dopo 2 minuti = errore!

- CORREZIONE = orologio più veloce

DEFLESSIONE DELLA LUCE

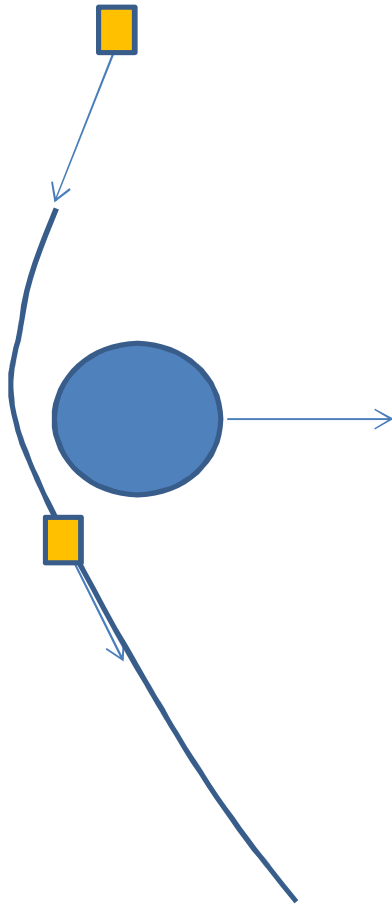
VOCABOLARIO

- Fionda gravitazionale
- Rimbalzo gravitazionale
- Biliardo cosmico (cosmic billiard)

- Fly-by
- Swing-by
- Gravity assist

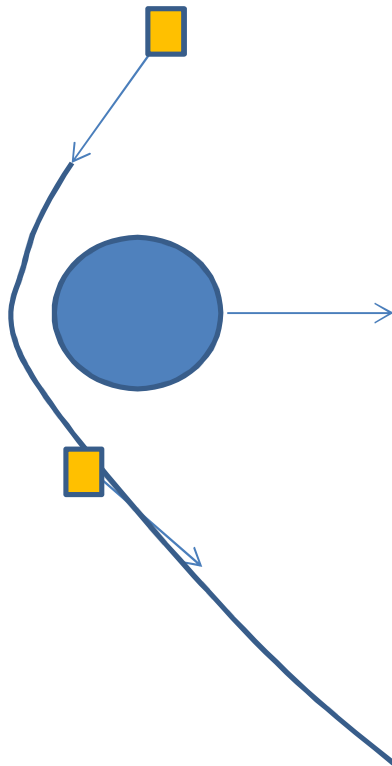
FIONDA GRAVITAZIONALE

(Fly by – cosmic billiard – swing by)



$$\Delta\varphi = \frac{2GM}{c^2 r_{MIN}}$$

DEVIAZIONE DI UN FOTONE NELLA RELATIVITA' GENERALE (Fly by – cosmic billiard – swing by)



$$\Delta\varphi = \frac{4GM}{c^2 r_{MIN}}$$

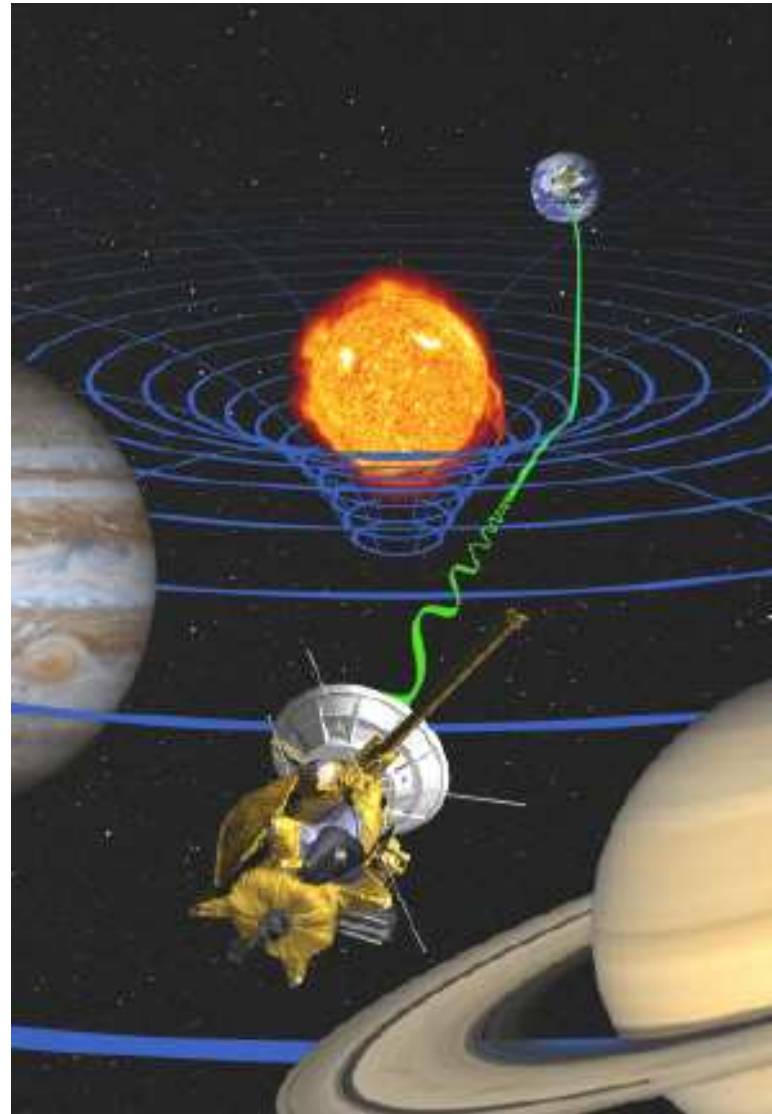
DEFLESSIONE DOPPIA RISPETTO
ALLA MECCANICA CLASSICA!

Cassini radio-science experiment (2002)

B. Bertotti

L. Iess

P. Tortora



Cassini 10 Years at Saturn BY THE NUMBERS

2 MILLION
COMMANDS
executed

2 BILLION
MILES TRAVELED
since arrival

514 
SCIENCE DATA
collected

3039
SCIENCE PAPERS
published

7 MOONS
discovered

206 ORBITS
completed

132 CLOSE
FLYBYS
of Saturn's moons

332,000
images taken

scientists from
26 NATIONS
participating

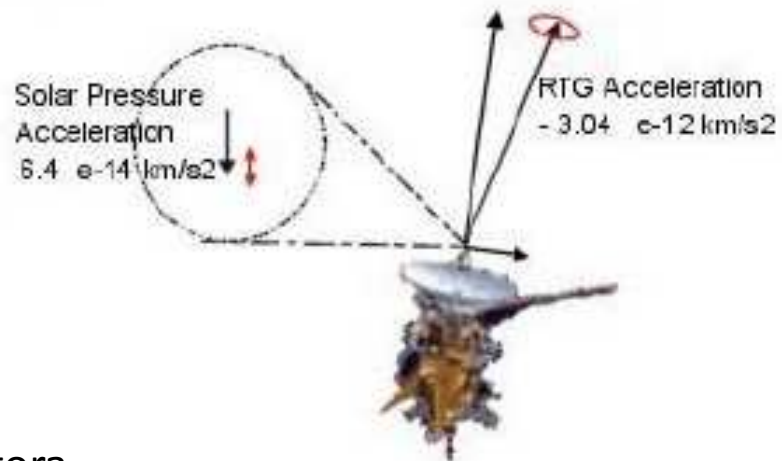
291 ENGINE
burns

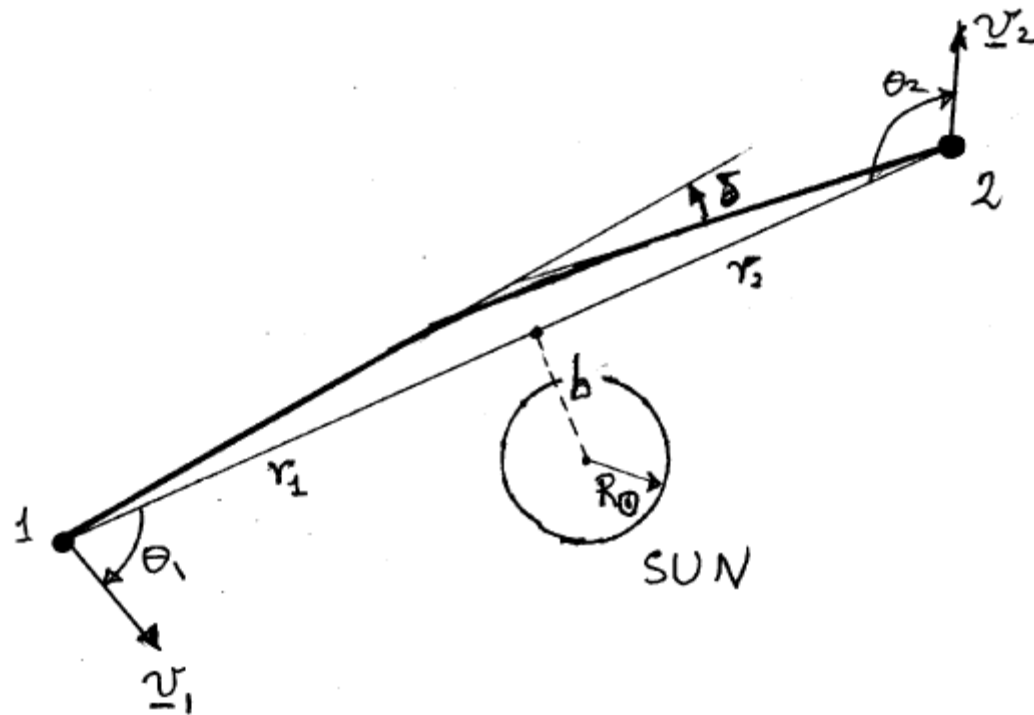


10
YEARS AT SATURN

- Effetto Doppler relativistico = cambio di fase delle onde radio, dovuto al passaggio vicino alla massa del Sole
- V(SONDA)
 - Accelerazioni Newtoniane
 - Accelerazioni non-Newtoniane
 - Pressione Solare
 - RTG

Non-gravitational acceleration in the orbital plane





Cassini's target accuracy (in K_a band):

$$\Delta v/v = 10^{-14} \text{ at } 1000 - 10,000 \text{ s (conjunctions)}$$

$$\Delta v/v = 3 \times 10^{-15} \text{ at } 1000 - 10,000 \text{ s (oppositions)}$$

$\Delta v/v = 10^{-14}$ corresponds to an accuracy in velocity of $3 \times 10^{-4} \text{ cm/s} !!$

Dynamic range of $10^{10} !!$

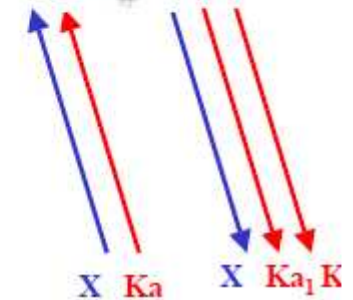


$$\gamma = 1 + 10^{-5}$$

Cassini spacecraft with high-gain antenna (points to the Earth)



observables

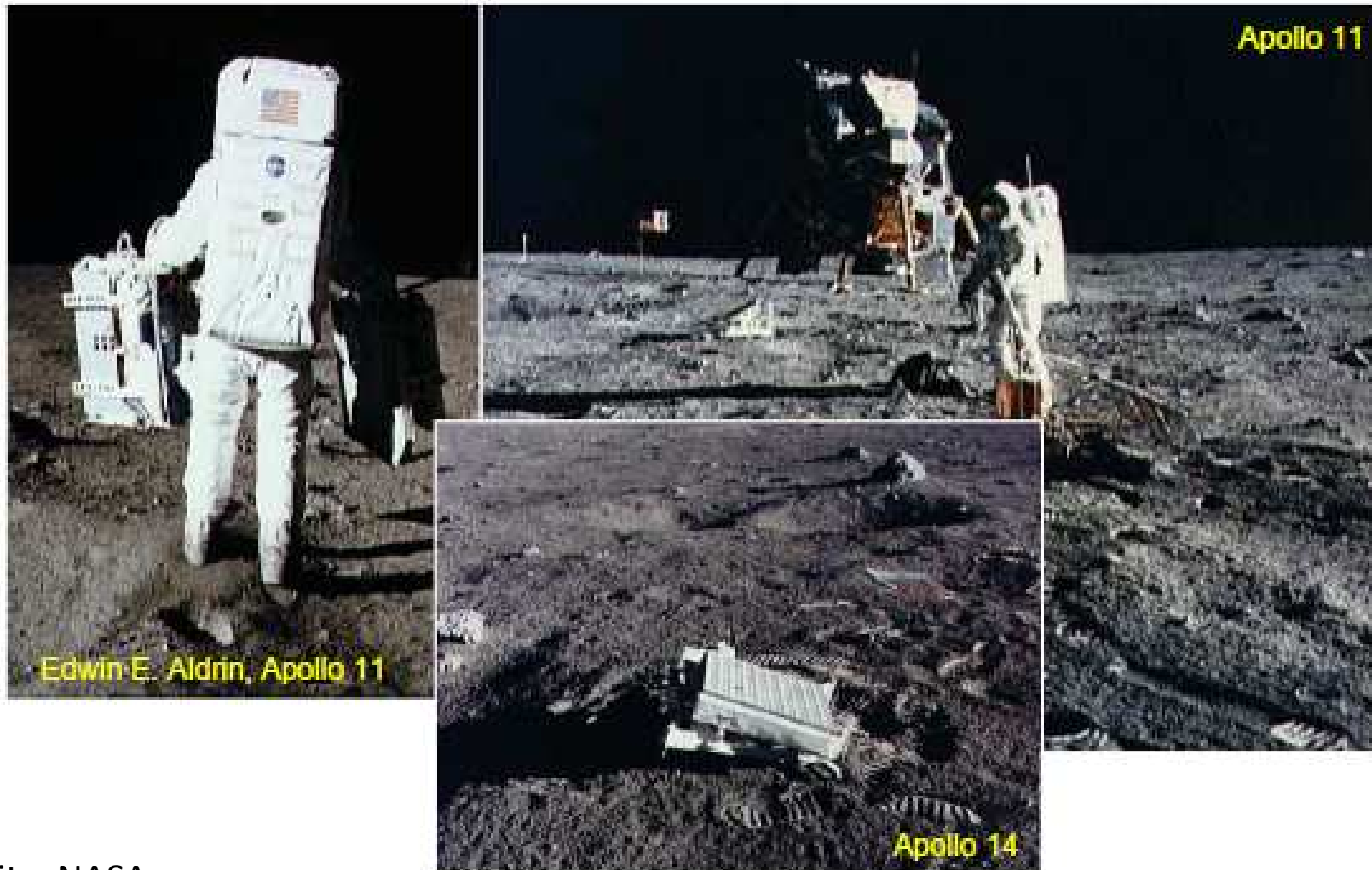


S-band 2-3 GHz
 X-band 7-8 GHz
 Ka-band 32-34 GHz

Effetti relativistici sull'orbita lunare

Excellent Legacy of the Apollo Program

The Apollo 11 retroreflector initiated a shift from analyzing lunar position angles to ranges. Today LLR is the **only** continuing experiment since the Apollo-Era.

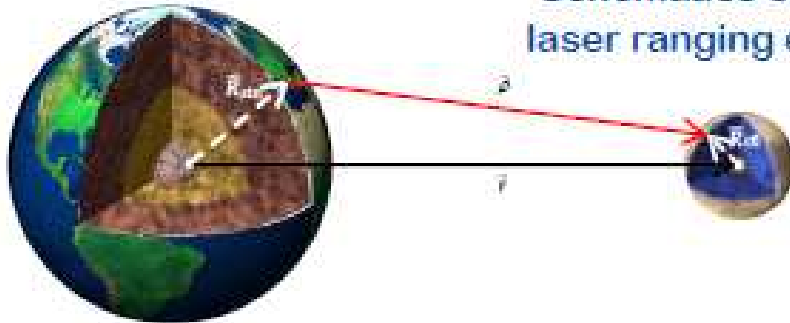




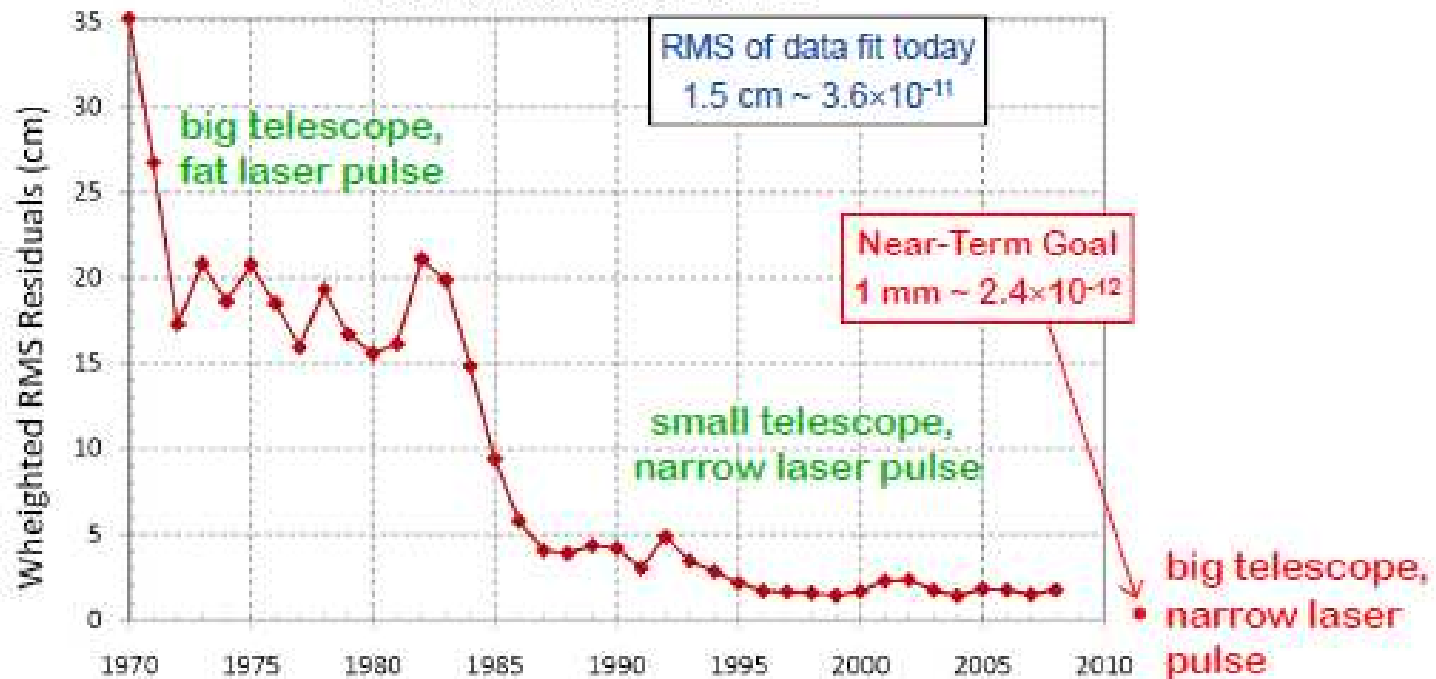


LUNAR LASER RANGING SCIENCE

Historical Accuracy of LLR



Historical Accuracy of LLR Fits





Largest Effects in Lunar Orbit

Largest Radial Amplitudes by Cause

Cause	Amplitude
Ellipticity	20,905 & 570 km
Solar perturbations	3,699 & 2,956 km
Jupiter perturbation	1.06 km
Venus perturbations	0.73, 0.68 & 0.60 km
Earth J_2	0.46 & 0.45 km
Moon J_2 & C_{22}	0.2 m
Earth C_{22}	0.5 mm
Solar radiation pressure	4 mm

Relativistic Effects on Orbit

Cause	Amplitude
Lorentz contraction	0.95 m
Solar potential	6 cm
Time transformation	5 & 5 cm
Other relativity	5 cm

Sources: Chapront-Touzé and Chapront, Vokrouhlicky, Williams and Dickey

TAU (1987)



A solar system escape velocity of 106 km/s is needed to propel the TAU vehicle to 1000 AU in 50 years. The NEP system must accelerate the vehicle for about ten years before this velocity is attained because of the extremely low thrust nature of the xenon-fueled ion engines. At the end of the thrusting phase the NEP system is jettisoned to allow the TAU spacecraft and science experiments to coast out to 1000 AU.